CHAPTER 8

Optimum filtering for transmission and reception

8.1 Introduction

There are two signal filtering techniques which are of basic importance in digital communications. The first is concerned with filtering for transmission in order to minimise signal bandwidth. The second is concerned with filtering at the receiver in order to maximise the SNR at the decision instant (and consequently minimise the probability of symbol error). This chapter examines each of these problems and establishes criteria to be met by those filters providing optimum solutions.

8.2 Pulse shaping for optimum transmissions

Spectral efficiency, η_s, is defined as the rate of information transmission per unit of occupied bandwidth[1], i.e.:

$$\eta_s = R_s H / B \quad \text{(bits/s/Hz)} \tag{8.1}$$

where R_s is the symbol rate, H is entropy, i.e. the average amount of information (measured in bits) conveyed per symbol, and B is occupied bandwidth. (For an alphabet containing M, statistically independent, equiprobable symbols, $H = \log_2 M$ bit/symbol, see Chapter 9.) The same term is also sometimes used for the quantity R_s/B which has units of symbol/s/Hz or baud/Hz. Since spectrum is a limited resource it is often desirable to minimise the bandwidth occupied by a signal of given baud rate. Nyquist's sampling theorem (section 5.3.2) limits the transmission rate of independent samples (or symbols) in a baseband bandwidth B to:

[1] In cellular radio applications the term spectral efficiency is also used in a more general sense, incorporating the spatial spectrum 'efficiency'. This quantity is variously ascribed units of voice channels/MHz/km^2, Erlangs/MHz/km^2 or voice channels/cell. (Typically these spectral 'efficiencies' are much greater than unity!) The quantity called spectral efficiency here is then referred to as bandwidth efficiency.

$$R_s \leq 2B \quad \text{(symbol/s)} \tag{8.2}$$

The essential pulse shaping problem is therefore one of how to shape transmitted pulses to allow signalling at, or as close as possible to, the maximum (Nyquist) rate of $2B$ symbol/s.

8.2.1 Intersymbol interference (ISI)

Rectangular pulse signalling, in principle, has a spectral efficiency of 0 bit/s/Hz since each rectangular pulse, strictly speaking, has infinite bandwidth. In practice, of course, rectangular pulses can be transmitted over channels with finite bandwidth if a degree of distortion can be tolerated.

In digital communications it might appear that distortion is unimportant since a receiver must only distinguish between pulses which have been distorted in the same way. This might be thought especially true for OOK signalling in which only the presence or absence of pulses is important. This is not, in fact, the case since, if distortion is severe enough, then pulses may overlap in time. The decision instant voltage might then arise not only from the current symbol pulse but also from one or more preceding pulses. The smearing of one pulse into another is called *intersymbol interference* and is illustrated in Figure 8.1 for the case of rectangular baseband pulses, distorted by an *RC* lowpass channel (as discussed previously in section 5.4).

8.2.2 Bandlimiting of rectangular pulses

The nominal bandwidth of a baseband, unipolar, NRZ signal with baud rate $R_s = 1/T_o$ symbol/s was taken in section 6.4.1 to be $B = 1/T_o$ Hz. (This corresponds to the positive frequency width of the signal's main spectral lobe.) It is instructive to see the effect of limiting a rectangular pulse to this bandwidth before transmission (Figure 8.2). The filtered pulse spectrum, Figure 8.2(f), is then restricted to the main lobe of the rectangular pulse spectrum, Figure 8.2(b). The filtered pulse shape, Figure 8.2(e), is the rectangular pulse convolved with the filter's sinc($2Bt$) impulse response. Figure 8.2(c) shows the rectangular pulse superimposed on the filter's impulse response for two values of time offset, 0 and T_o seconds. At zero offset the convolution integral gives the received pulse peak at $t = 0$. At an offset of $T_o/2$ (not shown) the convolution integral clearly gives a

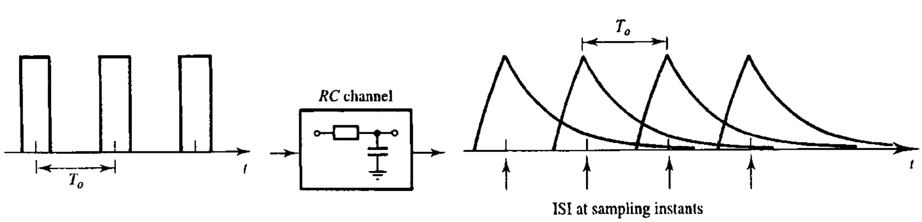


Figure 8.1 *Pulse smearing due to distortion in an RC channel.*

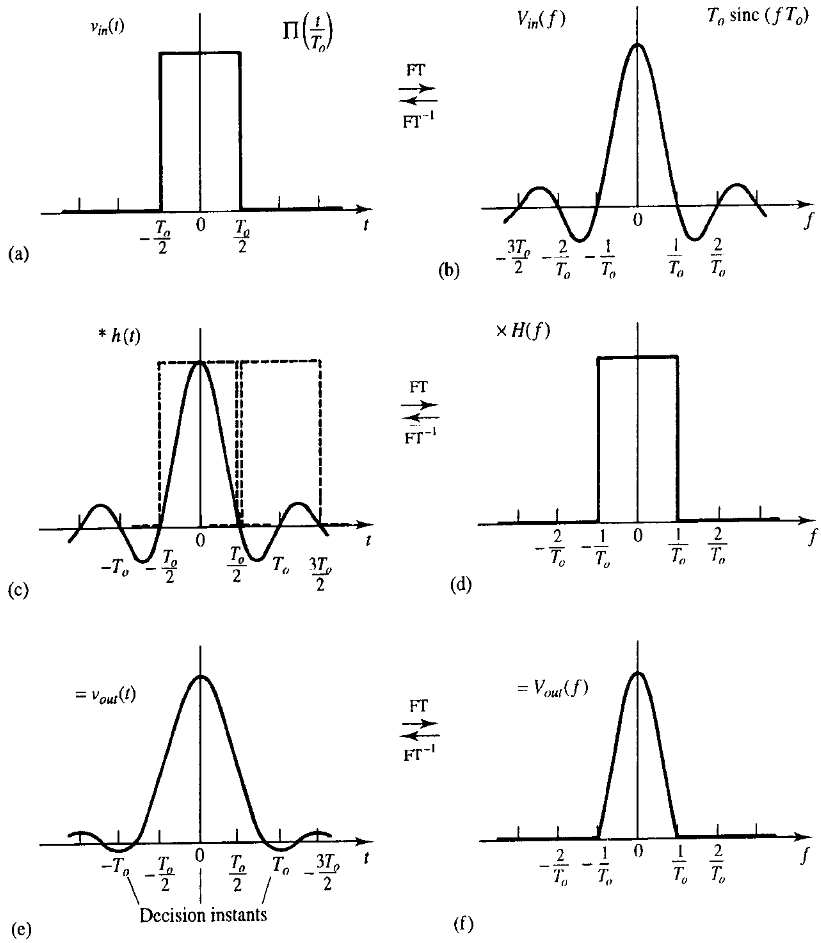


Figure 8.2 *NRZ rectangular pulse distortion due to rectangular frequency response filtering.*

reduced, but still large, positive result. At an offset of T_o the convolution gives a negative value (since the negative first sidelobe of the sinc function is larger than its positive second sidelobe). The zero crossing point of the filtered pulse therefore occurs a little before T_o seconds, Figure 8.2(e). At the centre point sampling instants $(\cdots, -T_o, 0, T_o, 2T_o, \cdots)$ receiver decisions would therefore be based not only on that pulse which should be considered but also, erroneously, on contributions from adjacent pulses. These unwanted contributions have the potential to degrade BER performance.

8.2.3 ISI free signals

The decision instants marked on Figure 8.2(e) illustrate an important point, i.e.:

Only decision instant ISI is relevant to the performance of digital communications systems. ISI occurring at times other than the decision instants does not matter.

If the signal pulses could be persuaded to pass through zero at every decision instant (except, of course, one) then ISI would no longer be a problem. This suggests a definition for an ISI free signal, i.e.:

An ISI free signal is any signal which passes through zero at all but one of the sampling instants.

Denoting the ISI free signal by $v_N(t)$ and the sampling instants by nT_o (where n is an integer and T_o is the symbol period) this definition can be expressed mathematically as:

$$v_N(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_o) = v_N(0)\, \delta(t) \tag{8.3}$$

The important property of ISI free signals, summarised by equation (8.3), is illustrated in Figure 8.3. Such signals suppress all the impulses in a sampling function except one (in this case the one occurring at $t = 0$). A good example of an ISI free signal is the sinc pulse, Figure 8.4. These pulses have a peak at one decision instant and are zero at all other decision instants as required. (They also have the minimum (Nyquist) bandwidth for a given baud rate.) An OOK, multilevel, or analogue PAM, system could, in principle, be implemented using sinc pulse signalling. In practice, however, there are two problems:

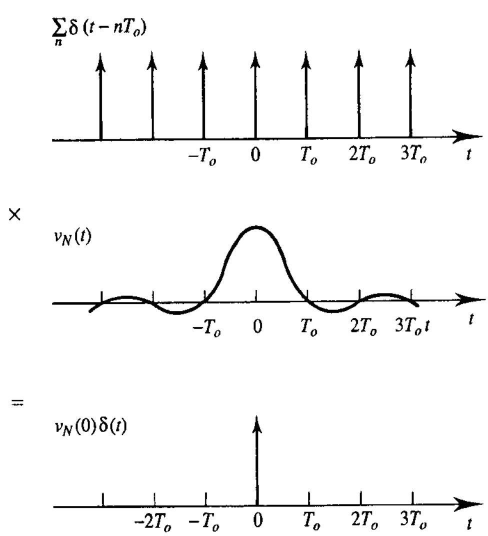
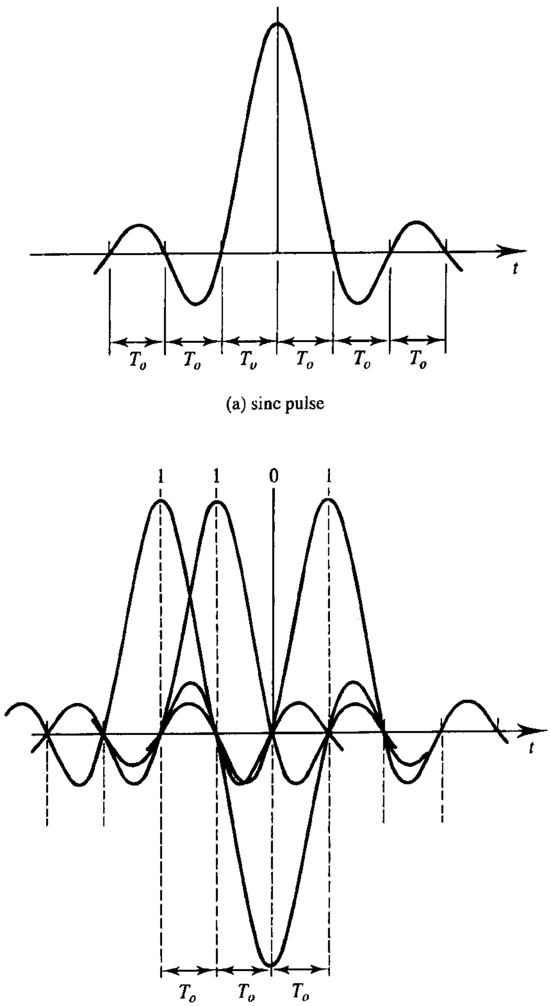


Figure 8.3 *Impulse suppression property of ISI free pulse.*



(a) sinc pulse

(b) sinc pulse signalling

Figure 8.4 *ISI free transmission using sinc pulses.*

1. sinc pulses are not physically realisable.
2. sinc pulse sidelobes (and their rates of change at the decision instants) are large and decay only with $1/t$.

The obvious way to generate sinc pulses is by shaping impulses with lowpass rectangular filters. The first problem could be equally well stated, therefore, as 'linear phase lowpass rectangular filters are not realisable'. The second problem means that extremely accurate decision timing would be required at the receiver to keep decision instant ISI to tolerable levels.

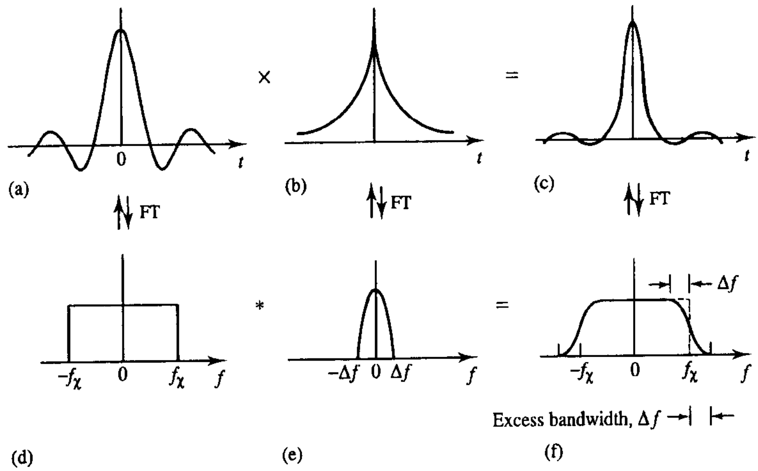


Figure 8.5 *Suppression of sinc pulse sidelobes and its effect on pulse spectrum.*

A more practical signal pulse shape would retain the desirable feature of sinc pulses (i.e. regularly spaced zero crossing points) but have an envelope with much more rapid roll-off. This can be achieved by multiplying the sinc pulse with a rapidly decaying monotonic function, Figure 8.5(a) to (c). In the frequency domain this corresponds to convolving the sinc pulse rectangular spectrum, Figure 8.5(d), with the spectrum of the decaying function, Figure 8.5(e), to obtain the final spectrum, Figure 8.5(f). As long as the decaying function is real and even its spectrum will be real and even which implies that the modified pulse spectrum will have *odd* symmetry about the sinc pulse's cut-off frequency, f_χ, Figure 8.5(f). This suggests an alternative definition for ISI free (baseband) signals, i.e.:

An ISI free baseband signal has a voltage spectrum which displays odd symmetry about $1/(2T_o)$ *Hz.*

A more general statement, which includes ISI free bandpass signals, can be made by considering frequency translation of the baseband spectrum using the modulation theorem (Figure 8.6), i.e.:

An ISI free signal has a voltage spectrum which displays odd symmetry between its centre frequency, f_c, *and* $f_c \pm 1/T_o$ *Hz.*

This property can also be demonstrated by Fourier transforming equation (8.3), replacing the product by a convolution (*), section 2.3.4, to give:

$$V_N(f) * \frac{1}{T_o} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_o}\right) = v_N(0) \tag{8.4}$$

which, using the replicating action of delta functions under convolution, gives:

$$\frac{1}{T_o} \sum_{n=-\infty}^{\infty} V_N\left(f - \frac{n}{T_o}\right) = v_N(0) \tag{8.5}$$

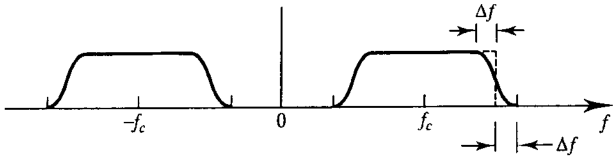


Figure 8.6 *Amplitude spectrum of a bandpass ISI free signal.*

Figure 8.7 is a pictorial interpretation of equation (8.5). The spectrum of the ISI free signal is such that if replicated along the frequency axis with periodic spacing $1/T_o$ Hz the sum of all replicas is a constant. This requires exactly the spectral symmetry described in the definitions given above.

EXAMPLE 8.1
Specify a baseband Nyquist channel which has a piecewise linear amplitude response, an absolute bandwidth of 10 kHz, and is appropriate for a baud rate of 16 kbaud. What is the channel's excess bandwidth?

The cut-off frequency of the parent rectangular frequency response is given by:

$$f_\chi = R_s/2 = 16 \times 10^3/2 = 8 \times 10^3 \text{ Hz}$$

The simplest piecewise linear roll-off therefore starts at 8 – 2 = 6 kHz, is 6 dB down at f_χ = 8 kHz and is zero (–∞ dB down) at 8 + 2 = 10 kHz. (The amplitude response, below the start of roll-off, is flat and the phase is linear.) Thus:

$$|H_N(f)| = \begin{cases} 1.0, & |f| < 6000 \text{ (Hz)} \\ 2.5 - 0.25 \times 10^{-3} f, & 6000 \le |f| \le 10000 \text{ (Hz)} \\ 0, & |f| > 10000 \text{ (Hz)} \end{cases}$$

The channel's excess bandwidth is 10 kHz –8 kHz = 2 kHz.

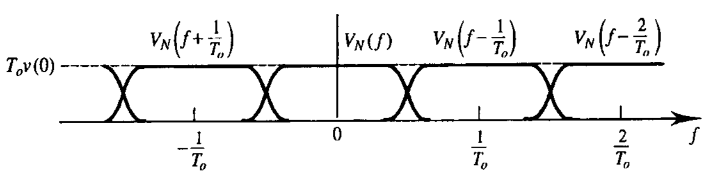


Figure 8.7 *Constant sum property of replicated ISI free signal spectra.*

8.2.4 Nyquist's vestigial symmetry theorem

Nyquist's vestigial symmetry theorem defines a symmetry condition on $H(f)$ which must be satisfied to realise an ISI free baseband impulse response. It can be stated as follows:

> *If the amplitude response of a lowpass rectangular filter with linear phase and bandwidth f_χ is modified by the addition of a real valued function having odd symmetry about the filter's cut-off frequency, then the resulting impulse response will retain at least those zero crossings present in the original sinc$(2f_\chi t)$ response, i.e. it will be an ISI free signal.*

This 'recipe' for deriving the whole family of Nyquist filters from a lowpass rectangular prototype is illustrated in Figure 8.8. The theorem requires no further justification since it follows directly from the spectral properties of ISI free signals (section 8.2.3). The theorem can be generalised to include filters with ISI free bandpass impulse responses in an obvious way.

8.2.5 Raised cosine filtering

The family of raised cosine filters is an important and popular subset of the family of Nyquist filters. The odd symmetry of their amplitude response is provided using a cosinusoidal, half cycle, roll-off (Figure 8.9). Their (lowpass) amplitude response therefore has the following piecewise form:

$$|H(f)| = \begin{cases} 1.0, & |f| \le (f_\chi - \Delta f) \\ \frac{1}{2}\left\{1 + \sin\left[\frac{\pi}{2}\left(1 - \frac{|f|}{f_\chi}\right)\frac{f_\chi}{\Delta f}\right]\right\}, & (f_\chi - \Delta f) < |f| < (f_\chi + \Delta f) \\ 0, & |f| \ge (f_\chi + \Delta f) \end{cases} \quad (8.6)$$

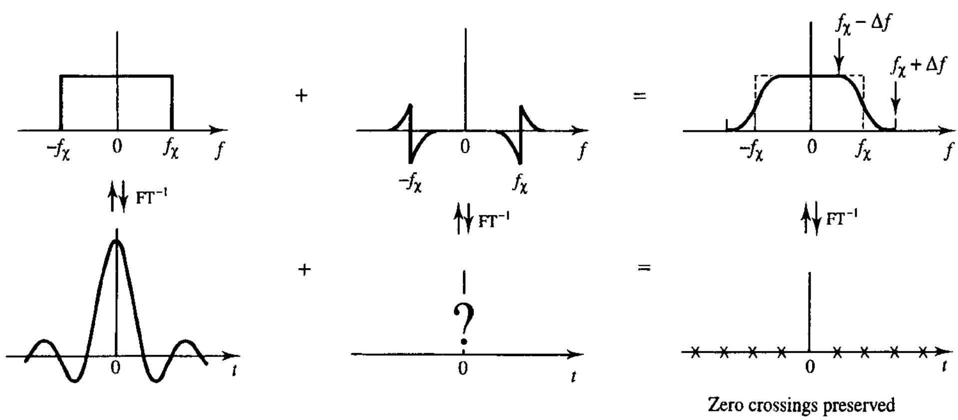


Figure 8.8 *Illustration of Nyquist's vestigial symmetry theorem.*

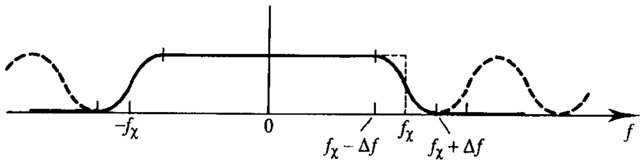


Figure 8.9 *Amplitude response of (linear phase) raised cosine filter ($f_\chi = R_s/2$).*

and their phase response is linear (implying the need for finite impulse response filters [Mulgrew and Grant]). f_χ in equation (8.6) is the cut-off frequency of the prototype rectangular lowpass filter (and the −6 dB frequency of the raised cosine filter). f_χ is related to the symbol period, T_o, by $f_\chi = R_s/2 = 1/(2T_o)$. Δf is the excess (absolute) bandwidth of the filter over the rectangular lowpass prototype. The normalised excess bandwidth, α, given by:

$$\alpha = \frac{\Delta f}{f_\chi} \tag{8.7}$$

is called the roll-off factor and can take any value between 0 and 1. Figure 8.10(a) shows the raised cosine amplitude response for several values of α. When $\alpha = 1$ the characteristic is said to be a *full* raised cosine and in this case the amplitude response simplifies to:

$$\begin{aligned} |H(f)| &= \begin{cases} \frac{1}{2}\left(1 + \cos\left(\frac{\pi f}{2f_\chi}\right)\right), & |f| \le 2f_\chi \\ 0, & |f| > 2f_\chi \end{cases} \\ &= \begin{cases} \cos^2\left(\frac{\pi f}{4f_\chi}\right), & |f| \le 2f_\chi \\ 0, & |f| > 2f_\chi \end{cases} \end{aligned} \tag{8.8}$$

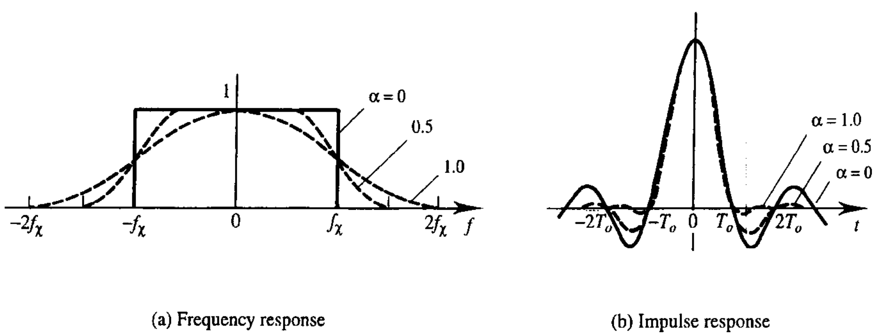


Figure 8.10 *Responses of raised cosine filters with three different roll-off factors.*

(The power spectral density (PSD) of an ISI free signal generated using a full raised cosine filter therefore has a $\cos^4[\pi f/(4f_\chi)]$ shape, Figure 8.11.) The impulse response of a full raised cosine filter (found from the inverse Fourier transform of equation (8.8)) is:

$$h(t) = 2f_\chi \frac{\sin 2\pi f_\chi t}{2\pi f_\chi t} \frac{\cos 2\pi f_\chi t}{1 - (4f_\chi t)^2} \tag{8.9}$$

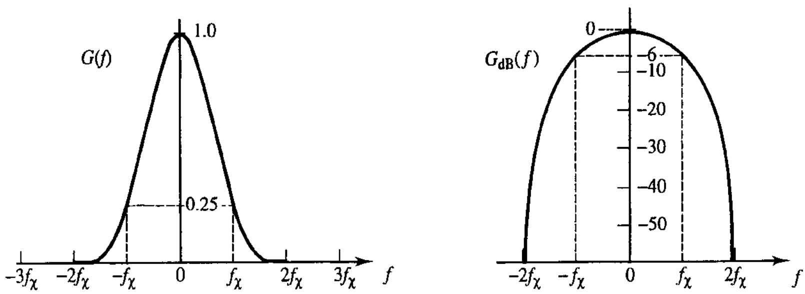
This is shown in Figure 8.10(b) along with the impulse responses of raised cosine filters with other values of α. The first part of equation (8.9) represents the sinc impulse response of the protype rectangular filter. The second part modifies this with extra zeros (due to the numerator) and faster decaying envelope ($1/t^3$ in total due to the denominator). The absolute bandwidth of a baseband filter (or channel) with a raised cosine frequency response is:

$$\begin{aligned} B &= \frac{1}{2T_o}(1+\alpha) \\ &= \frac{R_s}{2}(1+\alpha) \quad \text{(Hz)} \end{aligned} \tag{8.10}$$

where R_s is the symbol (or baud) rate. For a bandpass raised cosine filter the bandwidth is twice this, i.e.:

$$B = R_s(1+\alpha) \quad \text{(Hz)} \tag{8.11}$$

(This simply reflects the fact that when baseband signals are converted to bandpass (double sideband) signals by amplitude modulation, their bandwidth doubles.) Impulse signalling over a raised cosine *baseband* channel has a spectral efficiency of 2 symbol/s/Hz when $\alpha = 0$ and 1 symbol/s/Hz when $\alpha = 1$. For binary signalling systems (assuming equiprobable, independent, symbols) this translates to 2 bit/s/Hz and 1 bit/s/Hz respectively (see Chapter 9). For bandpass filters and channels these efficiencies are halved, due to the double sideband spectrum.



(a) Linear scale (normalised to spectral peak) (b) Logarithmic scale (dB with respect to spectral peak)

Figure 8.11 *PSD of an ISI free signal generated using a full raised cosine filter.*

EXAMPLE 8.2

What absolute bandwidth is required to transmit an information rate of 8.0 kbit/s using 64-level baseband signalling over a raised cosine channel with a roll-off factor of 40%?

The bit rate, R_b, is given by the number of bits per symbol (H) times the number of symbols per second (R_s), i.e.:

$$R_b = H \times R_s$$

Therefore:

$$\begin{aligned} R_s &= R_b/H \\ &= \frac{8 \times 10^3}{\log_2 64} = 1.333 \times 10^3 \text{ (symbol/s)} \\ B &= \frac{R_s}{2}(1+\alpha) \\ &= \frac{1.333 \times 10^3}{2}(1+0.4) \\ &= 933.1 \text{ (Hz)} \end{aligned}$$

This illustrates the bandwidth efficiency of a multi-level signal.

8.2.6 Nyquist filtering for rectangular pulses

It is possible to generate ISI free signals by shaping impulses with Nyquist filters. A more usual requirement, however, is to generate such signals by shaping rectangular pulses. The appropriate pulse shaping filter must then have a frequency response:

$$H(f) = \frac{V_N(f)}{\text{sinc}(\tau f)} \tag{8.12}$$

where $V_N(f)$ is the voltage spectrum of an ISI free pulse and $\text{sinc}(\tau f)$ is the frequency response of a hypothetical filter which converts impulses into rectangular pulses with width τ. If $V_N(f)$ is chosen to have a full raised cosine shape ($\alpha = 1$) then:

$$H(f) = \frac{\pi f \tau}{\sin(\pi f \tau)} \cos^2\left(\frac{\pi f}{4 f_\chi}\right) \tag{8.13}$$

Figure 8.12 ($\alpha = 1$) shows the frequency response corresponding to equation (8.13). Responses corresponding to other values of α are also shown.

8.2.7 Duobinary signalling

One of the problems associated with the use of sinc pulses, for ISI free signalling at the Nyquist rate, is the construction of a linear phase, lowpass rectangular filter. Such a filter

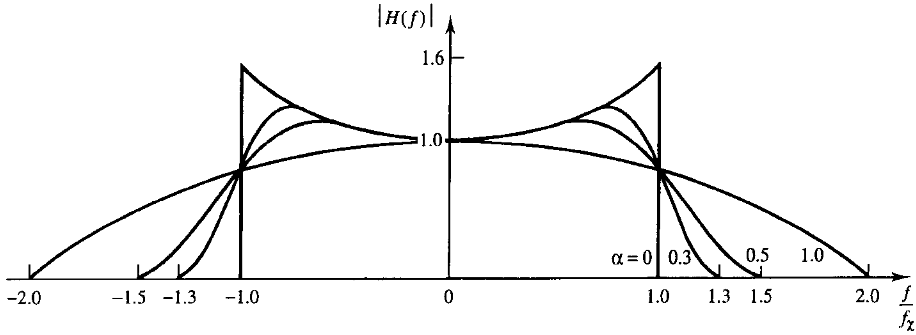


Figure 8.12 *Amplitude response of Nyquist filter for rectangular pulse shaping.*

(or rather an adequate approximation) is required to shape impulses. Duobinary signalling uses not a rectangular filter for pulse shaping but a cosine filter (not to be confused with the raised cosine filter). The amplitude response of a baseband cosine filter, Figure 8.13, is given by:

$$|H(f)| = \begin{cases} \cos \pi f T_o, & |f| \le 1/(2T_o) \\ 0, & |f| \le 1/(2T_o) \end{cases} \tag{8.14(a)}$$

and its (linear) phase response is usually taken to be:

$$\phi(f) = -\frac{\omega T_o}{2} \quad \text{(rad)} \tag{8.14(b)}$$

It has the same absolute bandwidth, $1/(2T_o)$ Hz, as the rectangular filter used for sinc signalling, and duobinary signalling therefore proceeds at the same maximum baud rate. Since its amplitude response has fallen to a low level at the filter band edge the linearity of the phase response in this region is not critical. This makes the cosine filter relatively easy to approximate. The impulse response of the cosine filter is most easily found by expressing its frequency response as a product of cosine, rectangular lowpass and phase factors, i.e.:

$$H(f) = \cos(\pi f T_o)\, \Pi\left(\frac{f}{2f_\chi}\right) e^{-j\pi f T_o} \tag{8.15}$$

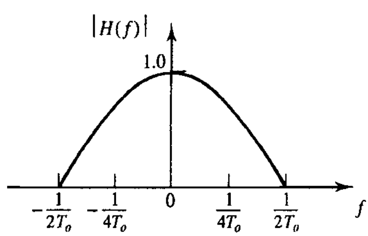


Figure 8.13 *Amplitude response of cosine filter.*

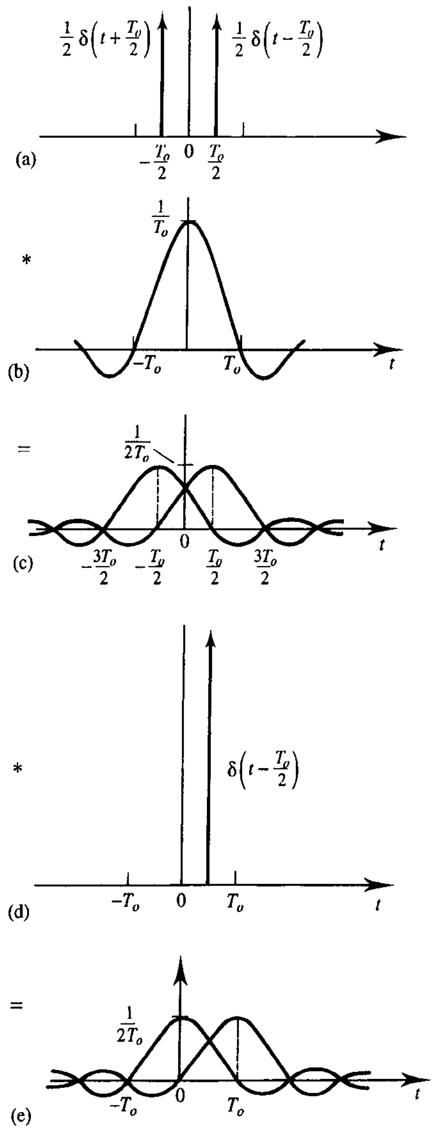


Figure 8.14 *Derivation of cosine filter impulse response.*

where $f_\chi = 1/(2T_o) = f_o/2$, and Π is the rectangular gate function. The product of the first two factors corresponds, in the time domain, to the convolution of a pair of delta functions with a sinc function, Figure 8.14(a) to (c). The third factor simply shifts the resulting pair of sinc functions to the right by $T_o/2$ seconds, Figure 8.14(d) and (e). The impulse response, shown in Figure 8.15, is therefore:

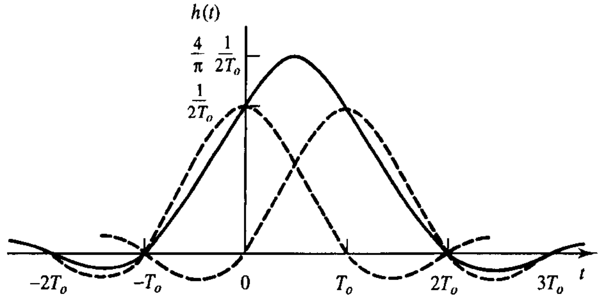


Figure 8.15 *Impulse response (solid curve) for the cosine filter.*

$$h(t) = \frac{1}{2T_o} [\text{sinc } f_o t + \text{sinc } f_o(t - T_o)] \tag{8.16}$$

It can be seen that the more gradual roll-off of the cosine filter (when compared to the rectangular filter) has been obtained at the expense of a significantly lengthened impulse response, which results in severe ISI. Duobinary signalling is important, however, because sampling instant interference occurs between *adjacent* symbols only, Figure 8.16, and is of predictable magnitude.

A useful model of duobinary signalling (which does *not* correspond to its normal implementation) is suggested by equation (8.16). It is clear that this is a superposition of two lowpass rectangular filter impulse responses, one delayed with respect to the other by T_o seconds. The cosine filter could therefore be implemented using a one symbol delay device, an adder and a rectangular filter, Figure 8.17. This implementation is obvious if equation (8.15) is rewritten as:

$$H(f) = \tfrac{1}{2}(1 + e^{-j2\pi f T_o})\, \Pi\left(\frac{f}{2f_\chi}\right) \tag{8.17}$$

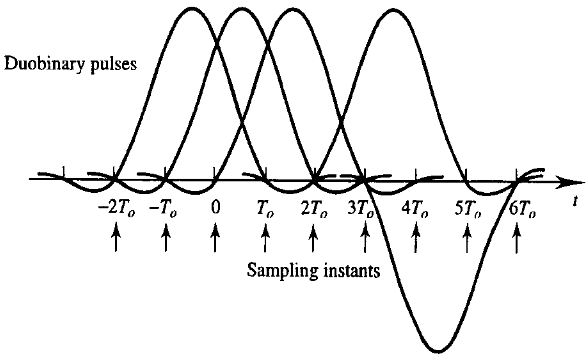


Figure 8.16 *Adjacent symbol ISI for duobinary signalling.*

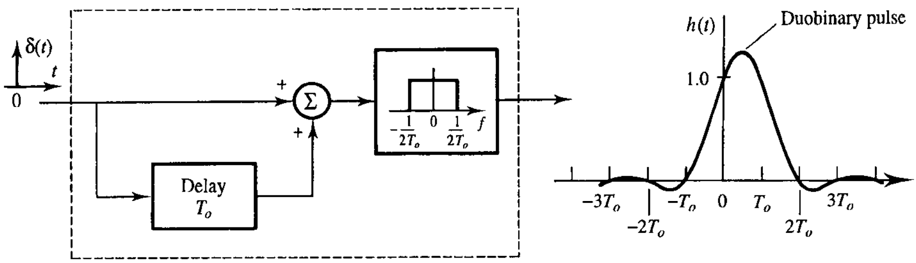


Figure 8.17 *Equivalent model of cosine filter for duobinary signalling.*

Duobinary signalling can therefore be interpreted as adjacent pulse summation followed by rectangular lowpass filtering. Figure 8.18 shows the composite pulses which arise from like and unlike combinations of binary impulse pairs. At the receiver, sampling can take place at the centre of the composite pulses (i.e. at the point of maximum ISI mid-way between the response peaks of the original binary impulses). This results in *three* possible levels at each decision instant, i.e. $+V$, 0 and $-V$, the level observed depending on whether the binary pulse pair are both positive, both negative or of opposite sign, Figure 8.19. Like bipolar line coding (Chapter 6), duobinary signalling is therefore a form of pseudo-ternary signalling [Lender]. The summing of adjacent pulse pairs at the transmitter can be described explicitly using the notation:

$$z_k = y_k + y_{k-1} \tag{8.18}$$

where z_k represents the k^{th} (ternary) symbol after duobinary coding and y_k represents the k^{th} (binary) symbol before coding. The decoding process after detection at the receiver is therefore the inverse of equation (8.18), i.e.:

$$\hat{y}_k = \hat{z}_k - \hat{y}_{k-1} \tag{8.19}$$

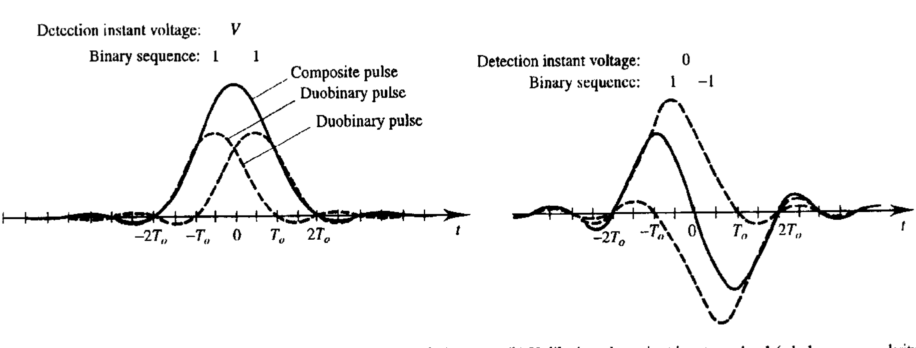


Figure 8.18 *Composite pulses arising from like and unlike combinations of input impulse pairs.*

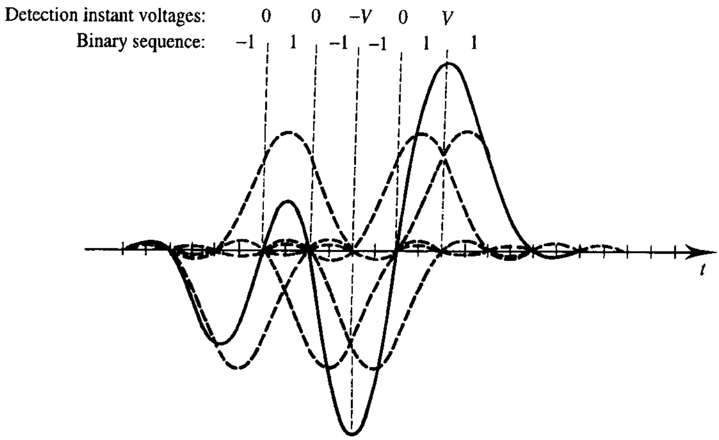
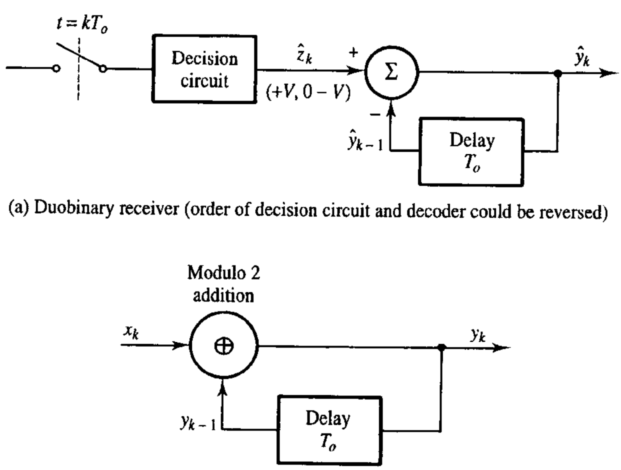


Figure 8.19 *Duobinary waveform arising from an example binary sequence.*

(where the hats (^) distinguish between detected and transmitted symbols to allow for the possibility of errors). The block diagram of a duobinary receiver is shown in Figure 8.20(a).

The essential advantage of duobinary signalling is that it permits signalling at the Nyquist rate without the need for linear phase, rectangular, lowpass filters (or their equivalent). The disadvantages are:

1. The ternary nature of the signal requires approximately 2.5 dB greater SNR when compared with ideal binary signalling for a given probability of error.



(a) Duobinary receiver (order of decision circuit and decoder could be reversed)

(b) Precoding for duobinary signalling

Figure 8.20 *Duobinary receiver and precoder.*

2. There is no one to one mapping between detected ternary symbols and the original binary digits.
3. The decoding process, $\hat{y}_k = \hat{z}_k - \hat{y}_{k-1}$, results in the propagation of errors.
4. The duobinary power spectral density (which is $\cos^2(\pi T_o f)\,\Pi[f/(2f_\chi)]$, i.e. the square of the cosine filter amplitude response) has a maximum at 0 Hz making it unsuitable for use with AC coupled transmission lines.

Problems 2 and 3 can be solved by adding the following precoding algorithm to the bit stream prior to duobinary pulse shaping:

$$y_k = x_k \oplus y_{k-1} \tag{8.20}$$

where y_k is the k^{th} precoded bit, x_k is the uncoded bit and $\oplus$ represents modulo 2 addition. Figure 8.20(b) shows the block diagram corresponding to equation (8.20). The effect of precoding plus duobinary coding can be simplified, i.e.:

$$\begin{aligned} z_k &= (x_k \oplus y_{k-1}) + (x_{k-1} \oplus y_{k-2}) \\ &= (x_k \oplus y_{k-1}) + y_{k-1} \end{aligned} \tag{8.21}$$

The truth table for equation (8.21) is shown in Table 8.1. The important property of this table is that $z_k = 1$ when, and only when, $x_k = 1$. The precoded duobinary signal can therefore be decoded on a bit by bit basis (i.e. without the use of feedback loops). One to one mapping, of received ternary symbols to original binary symbols, is thus re-established and error propagation is eliminated.

Table 8.1 *Precoded duobinary truth table.*

x_k	y_{k-1}	$x_k \oplus y_{k-1}$	z_k
0	0	0	0
1	0	1	1
0	1	1	2
1	1	0	1

EXAMPLE 8.3

Find the output data sequence of a duobinary signalling system (a) without precoding and (b) with precoding if the input data sequence is: 1 1 0 0 0 1 0 1 0 0 1 1 1.

(a) Using equation (8.18):

input data, y_k 1100010100111

y_{k-1} ?1100010100111

duobinary data, z_k ?2100111101222?

(b) Using equations (8.20) and (8.21) or Table 8.1, and assuming that the initial precoded bit is a digital 1:

x_k	1100010100111
y_{k-1}	(1)011110011101
z_k	1122210122111

Assuming that, after precoding, the (y_k) binary digits (1,0) are represented by positive and negative impulses, then the duobinary detection algorithm is:

$$\hat{x}_k = \begin{cases} 1, & f(kT_s) = 0 \\ 0, & f(kT_s) = \pm V \end{cases} \tag{8.22}$$

where $f(kT_s)$ is the received voltage at the appropriate decision instant.

Problem 4 (i.e. the large DC value of the duobinary PSD) can be addressed by replacing the transmitter one bit delay and adder in Figure 8.17 by a two bit $(2T_o)$ delay and subtractor, Figure 8.21(a). This results in *modified duobinary* signalling. The PSD has a null at 0 Hz but is still strictly bandlimited to $1/(2T_o)$ Hz. Figure 8.21(b) is a block diagram of the modified duobinary receiver. The frequency response of the pulse shaping filter, Figure 8.22(a), is:

$$H(f) = ½\,(1 - e^{-j4\pi f T_o})\,\Pi\left(\frac{f}{2f_\chi}\right) \tag{8.23}$$

where, once again, the filter cut-off frequency is $f_\chi = 1/(2T_o)$ Hz. The corresponding

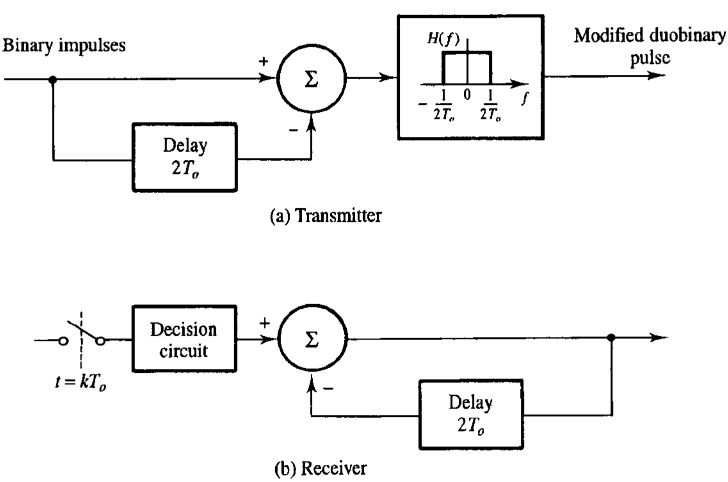


Figure 8.21 *Equivalent model for modified duobinary signalling.*

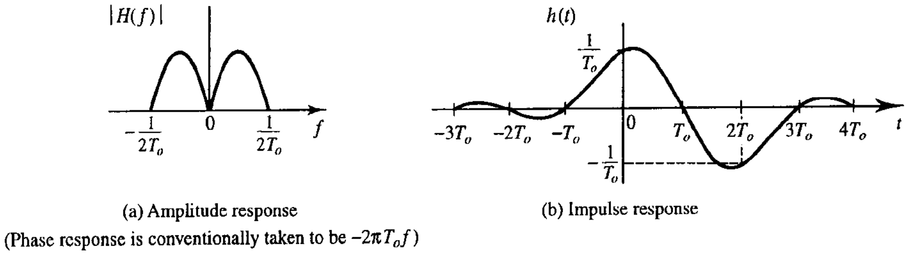


Figure 8.22 *Characteristics of pulse shaping filter for modified duobinary signalling.*

impulse response is shown in Figure 8.22(b). The amplitude and phase response are:

$$|H(f)| = \begin{cases} |\sin(2\pi f T_o)|, & |f| \leq 1/(2T_o) \\ 0, & |f| > 1/(2T_o) \end{cases} \tag{8.24}$$

$$\phi(f) = -\omega T_o, \qquad |f| \leq 1/(2T_o) \tag{8.25}$$

Since $(1 - e^{-j4\pi f T_o})$ can be factorised into $(1 - e^{-j2\pi f T_o})\,(1 + e^{-j2\pi f T_o})$, modified duobinary pulse shaping can be realised by cascading a one bit delay and subtractor (to implement the first factor) with a conventional duobinary pulse shaping filter (to implement the second factor). The block diagram corresponding to this implementation is shown in Figure 8.23. Precoding can be added to avoid modified duobinary error propagation. The appropriate precoding and post-decoding algorithms are illustrated as block diagrams in Figure 8.24.

8.2.8 Partial response signalling

Partial response signalling is a generalisation of duobinary signalling in which the single element transversal filter of Figure 8.17 is replaced with an N-element (tap weighted) filter. This produces a multilevel signal with non-zero correlation between symbols over an $N + 1$ symbol window. Since the ISI introduced as a result of this correlation is of a prescribed form it can be, as in duobinary signalling, effectively cancelled at the receiver. Partial response signalling is also known as correlative coding.

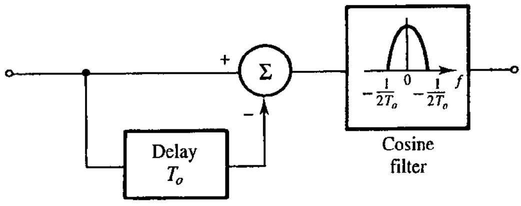


Figure 8.23 *Practical implementation of modified duobinary pulse shaping filter.*

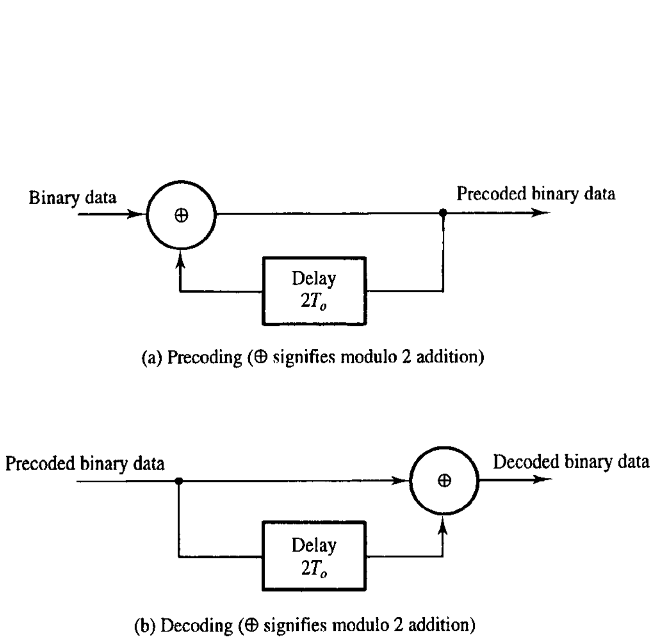


Figure 8.24 *Precoding and de-precoding for modified duobinary signalling.*

8.3 Pulse filtering for optimum reception

Formulas were derived in sections 6.2.1 and 6.2.2 for the probability of bit error expected when equiprobable, rectangular, baseband symbols are detected, using a centre point decision process in the presence of Gaussian noise. Since this process compares a single sample value of signal plus noise with an appropriate threshold the following question might be asked. *If several samples of the signal plus noise voltage are examined at different time instants within the duration of a single symbol (as illustrated in Figure 8.25) is it not possible to obtain a more reliable (i.e. lower P_e) decision?* The answer to this question is normally yes since, at the very least, majority voting of multiple decisions associated with a given symbol could be employed to reduce the probability of error. Better still, if n samples were examined, an obvious strategy would be to add the samples together and compare the result with n times the appropriate threshold for a single sample. If this idea is extended to its limit (i.e. $n \rightarrow \infty$) than the discrete summation of symbol plus noise samples becomes continuous integration of the symbol plus noise

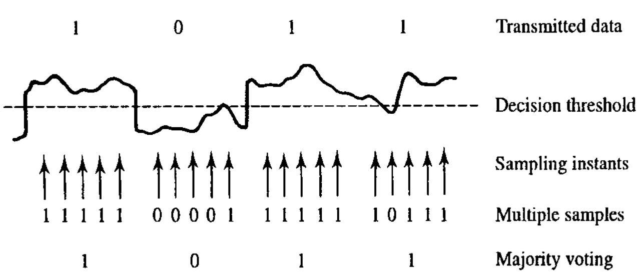


Figure 8.25 *Multiple sampling of single symbols.*

voltage. The post integration decision threshold then becomes ½ ($\int_0^{T_o} v_0\, dt + \int_0^{T_o} v_1\, dt$) where v_0 and v_1 are the voltage levels representing binary zeros and ones respectively. After each symbol the integrator output would be reset to zero ready for the next symbol. This signal processing technique, Figure 8.26, is a significant improvement on centre point sampling and is called integrate and dump (I+D) detection. It is the optimum detection process for baseband rectangular pulses in that the resulting probability of error is a minimum. It is also easy to implement as shown in Figure 8.27. I+D is a special case of a general and optimum type of detection process, which can be applied to any pulse shape, called *matched filtering*.

8.3.1 Matched filtering

A matched filter can be defined as follows:

A filter which immediately precedes the decision circuit in a digital communications receiver is said to be matched to a particular symbol pulse, if it maximises the output SNR at the sampling instant when that pulse is present at the filter input.

The criteria which relate the characteristics (amplitude and phase response) of a filter to those of the pulse to which it is matched can be derived as follows.

Consider a digital communications system which transmits pulses with shape $v(t)$, Figure 8.28(a). The pulses have a (complex) voltage spectrum $V(f)$, Figure 8.28(b) and a normalised energy spectral density (ESD) $|V(f)|^2$ V^2s/Hz, Figure 8.28(d). If the noise power spectral density (NPSD) is white, it can be represented as a constant ESD per pulse period as shown in Figure 8.28(d). If the spectrum is divided into narrow frequency

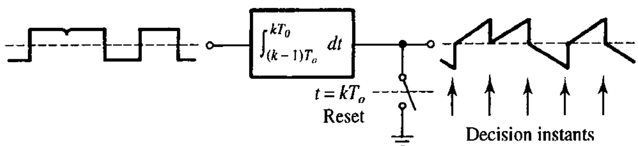


Figure 8.26 *Integrate and dump detection for rectangular pulses.*

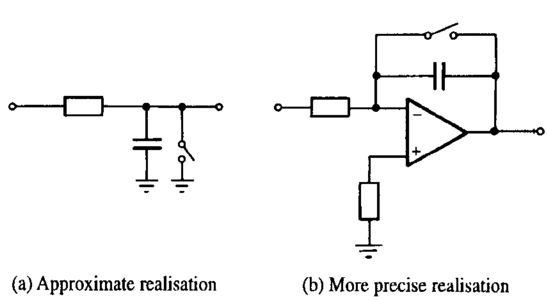


Figure 8.27 *Simple circuit realisations for integrate and dump (I+D) detection.*

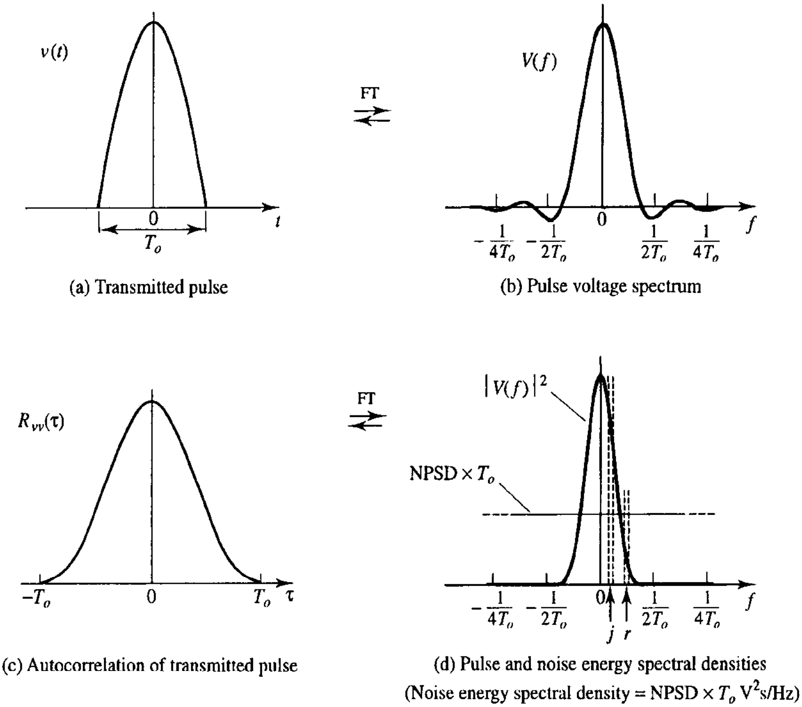


Figure 8.28 *Relationship between energy spectral densities of signal pulse and white noise to illustrate matched filtering amplitude criterion.*

bands it can be seen from this figure that some bands (such as j) have large SNR and some (such as r) have much smaller SNR. Any band which includes signal energy should clearly make a contribution to the decision process (otherwise signal is being discarded). It is intuitively obvious, however, that those bands with high SNR should be correspondingly more influential in the decision process than those with low SNR. This suggests forming a weighted sum of the individual sub-band signal and noise energies where the weighting is in direct proportion to each band's SNR. Since the NPSD is constant with frequency the SNR is proportional to $|V(f)|^2$. Remembering that the power or energy density passed by a filter is proportional to $|H(f)|^2$, this argument leads to the following statement of the amplitude response required for a matched filter assuming white noise:

The square of the amplitude response of a matched filter has the same shape as the energy spectral density of the pulse to which it is matched.

Now consider the pulse spectrum in Figure 8.28 to be composed of many closely spaced and harmonically related spectral lines (Figure 8.29(c), (d)). The amplitude and phase spectra give the amplitude and phase of each of the cosine waves into which a periodic version of the pulse stream has been decomposed (Figure 8.29(a), (b)). If it can

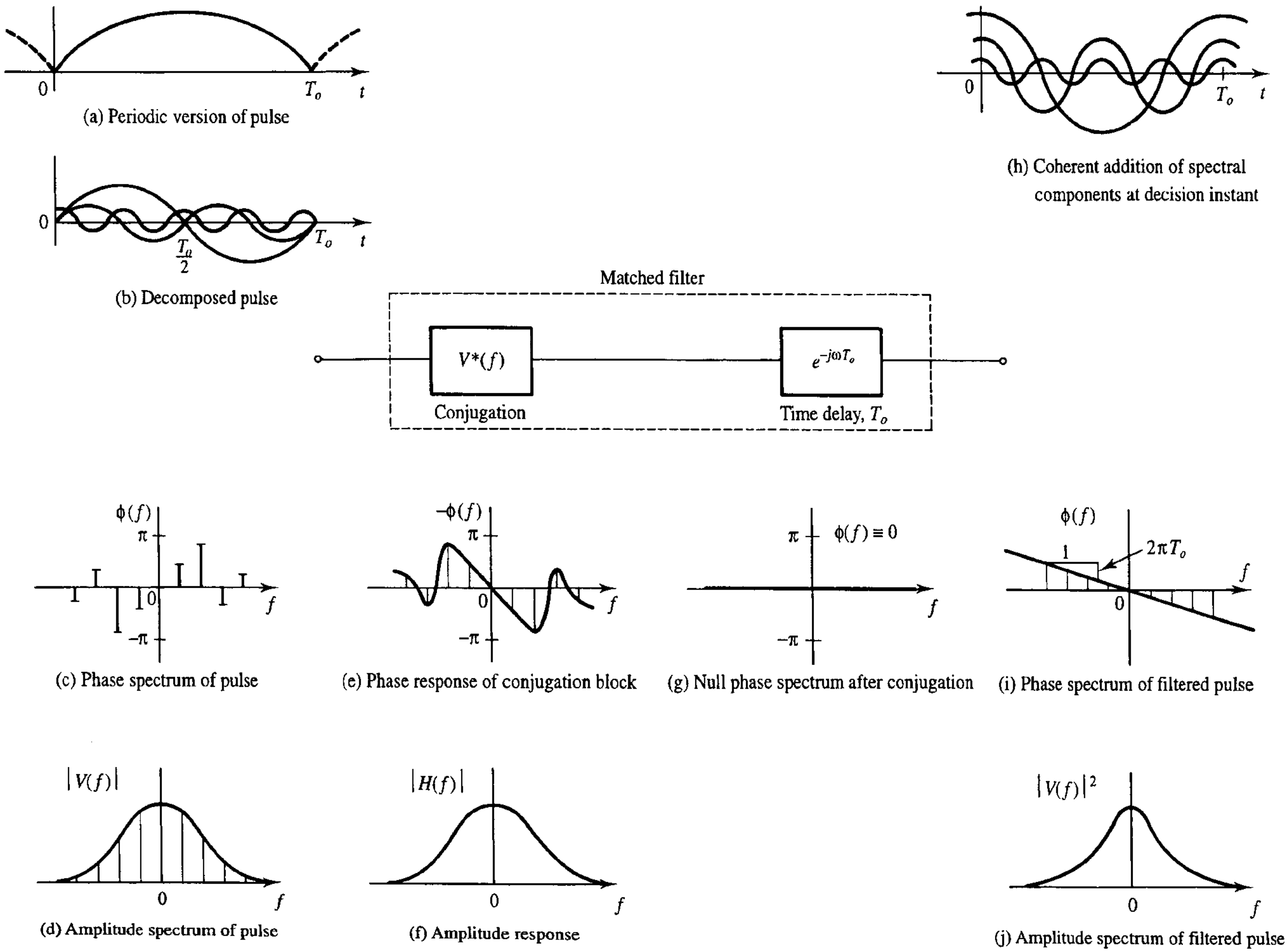


Figure 8.29 *Schematic illustration demonstrating origin of matched filtering phase criterion.*

be arranged for all the cosine waves to reach a peak simultaneously in time then the signal voltage (and therefore the signal power) will be a maximum at that instant.

The filter which achieves this has a phase response which is the opposite (i.e. the negative) of the pulse phase spectrum, Figure 8.29(e). The post-filtered pulse would then have a null phase spectrum, Figure 8.29(g), and the component cosine waves would peak together at time $t = 0, T_o, 2T_o, \cdots$ (Figure 8.29(h)). In practice a linear phase shift, $e^{-j\omega T_o}$, corresponding to a time delay of T_o seconds, must be added (or rather included) in the matched filter's frequency response to make it realisable[1]. This gives us a statement of the phase response required for a matched filter, i.e.:

The phase response of a matched filter is the negative of the phase spectrum of the pulse to which it is matched plus an additional linear phase of $-2\pi f T_o$ *rad.*

The matched filtering amplitude and phase criteria can be expressed mathematically as:

$$|H(f)|^2 = k^2 |V(f)|^2 \tag{8.26(a)}$$

$$\phi(f) = -\phi_v(f) - 2\pi T_o f \text{ (rad)} \tag{8.26(b)}$$

where $\phi_v(f)$ is the phase spectrum of the expected pulse and k is a constant. Equations (8.26) can be combined into a single matched filtering criterion, i.e.:

$$H(f) = kV^*(f)\, e^{-j\omega T_o} \tag{8.27}$$

where the superscript * indicates complex conjugation.

Matched filtering essentially takes advantage of the fact that the pulse or signal frequency components are coherent in nature whilst the corresponding noise components are incoherent. It is therefore possible, using appropriate processing, to add spectral components of the signal voltage-wise whilst the same processing adds noise components only power-wise. The extension of the above arguments to pulses buried in non-white noise is straightforward in which case the matched filtering amplitude response generalises to:

$$|H(f)| = \frac{k|V(f)|}{\sqrt{G_n(f)}} \tag{8.28}$$

where $G_n(f)$ is noise power spectral density (see Chapter 3). The phase response is identical to that for white noise.

EXAMPLE 8.4

Find the frequency response of the filter which is matched to the triangular pulse $\Lambda(t-1)$.

[1] This is to shift the *(single)* instant of constructive interference, between the (elemental) component sinusoids of the *(single)* aperiodic symbol, from $t = 0$ to $t = T_o$.

The voltage spectrum of the pulse is given by:

$$V(f) = \text{FT}\,\{\Lambda(t-1)\}$$
$$= \text{sinc}^2(f)\, e^{-j2\pi f 1}$$

The frequency response of the matched filter is therefore:

$$H(f) = V^*(f)\, e^{-j\omega T_o}$$
$$= \text{sinc}^2(f)\, e^{+j2\pi f}\, e^{-j2\pi f 2}$$
$$= \text{sinc}^2(f)\, e^{-j2\pi f}$$

8.3.2 Correlation detection

We now apply correlation (described in section 2.6) to receiver design. The impulse response of a filter is related to its frequency response by the inverse Fourier transform, i.e.:

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi ft}\, df \tag{8.29}$$

This equation can therefore be used to transform the matched filtering criterion described by equation (8.27) into the time domain:

$$h(t) = \int_{-\infty}^{\infty} k\, V^*(f)\, e^{j2\pi f(t-T_o)}\, df$$
$$= k \left[\int_{-\infty}^{\infty} V(f)\, e^{j2\pi f(T_o-t)}\, df \right]^* \tag{8.30}$$

i.e.:

$$h(t) = k\, v^*(T_o - t) \tag{8.31}$$

Equation (8.31) is a statement of the matched filtering criterion in the time domain. For a filter matched to a purely real pulse it can be expressed in words as follows:

> *The impulse response of a matched filter is a time reversed version of the pulse to which it is matched, delayed by a time equal to the duration of the pulse.*

Figure 8.30 illustrates equation (8.31) pictorially. The time delay, T_o, is needed to ensure causality (section 4.5) and corresponds to the need for the linear phase factor in equation (8.27).

Equation (8.31) allows the output pulse of a matched filter to be found directly from its input. The output of any time invariant linear filter is its input convolved with its impulse response. The convolution process involves reversing one of the functions

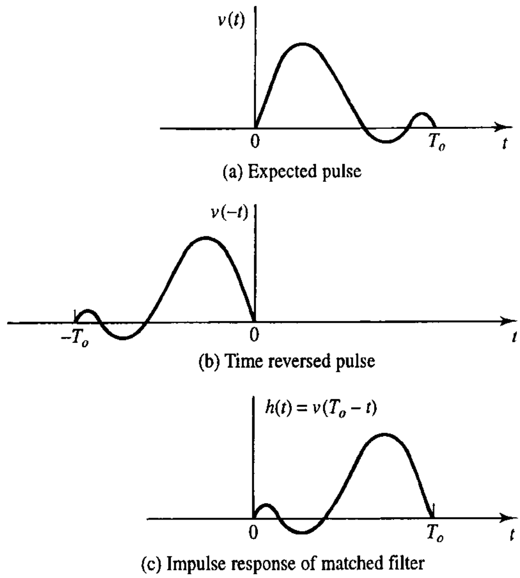


Figure 8.30 *Relationship between expected pulse and impulse response of matched filter.*

(either input or impulse response), sliding the reversed over the non-reversed function and integrating the product. Since the impulse response of the matched filter is a time reversed copy of the expected input, and since convolution requires a further reversal, then the output is given by the integrated sliding product of *either* the input *or* the impulse response with an *un*reversed version of *itself*. This is illustrated in Figure 8.31. The output is thus the *autocorrelation* (section 2.6) of either the input pulse *or* the impulse response. An algebraic proof for a real signal pulse is given below.

Let $v_{in}(t)$, $v_{out}(t)$ and $h(t)$ be the input, output and impulse response of a filter. Then by convolution (section 4.3.4):

$$v_{out}(t) = v_{in}(t) * h(t) \tag{8.32}$$

If the filter is matched to $v_{in}(t)$ then:

$$v_{out}(t) = v_{in}(t) * k\, v_{in}(T_o - t)$$

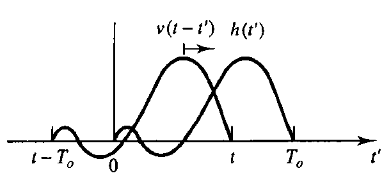


Figure 8.31 *Equivalence of* $v(t) * h(t)$ *and* $R_{hh}(t')$.

$$= k \int_{-\infty}^{\infty} v_{in}(t') \, v_{in}(T_o + t' - t) \, dt' \tag{8.33}$$

Putting $T_o - t = \tau$:

$$v_{out}(t) = v_{out}(T_o - \tau) = k \int_{-\infty}^{\infty} v_{in}(t') \, v_{in}(t' + \tau) \, dt'$$

$$= k \, R_{v_{in}v_{in}}(\tau)$$

$$(= k \, R_{hh}(\tau)) \tag{8.34}$$

Equation (8.34) can be expressed in words as follows:

The output of a filter driven by, and matched to, a real input pulse is, to within a multiplicative constant, k, and a time shift, T_o, the autocorrelation of the input pulse.

EXAMPLE 8.5

What will be the output of a filter matched to rectangular input pulses with width 1.0 ms?

The output pulse is the autocorrelation of the input pulse (equation (8.34)). Thus the output pulse will be triangular with width 2.0 ms.

The correlation property of a matched filter can be realised directly in the time domain. A block diagram of a classical correlator is shown in Figure 8.32. The correlator input pulse, $v_{in}(t)$, is distinguished from the reference pulse by a subscript since the input is strictly the sum of the signal pulse plus noise, i.e.:

$$v_{in}(t) = v(t) + n(t) \tag{8.35}$$

In digital communications the variable delay, τ, is usually unnecessary (Figure 8.33) since the pulse arrival times are normally known. Furthermore, it is only the peak value of the correlation function, $R_{v_{in}v}(0)$, which is of importance. The correlator output

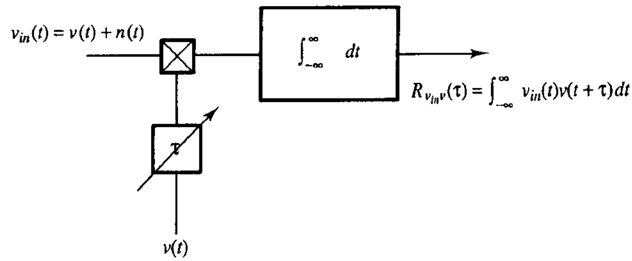


Figure 8.32 *Block diagram of signal correlator.*

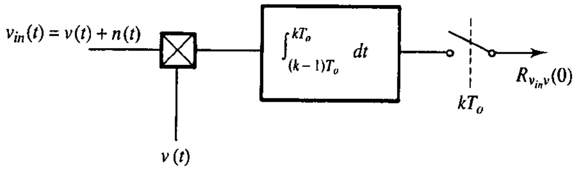


Figure 8.33 *Signal correlator for digital communications receiver.*

reaches this maximum value at the end of the input pulse, i.e. after T_o seconds. This, therefore, represents the correct sampling instant which leads to an optimum (i.e. maximum) decision SNR. (A matched filter response for a baseband coded waveform using the receiver correlation operation was demonstrated previously in Figure 4.9.)

It is interesting to note that an analogous argument to that used in the frequency domain to derive the amplitude response of a matched filter (equation (8.26(a)) can be used to demonstrate the optimum nature of correlation detection. Specifically, if the noise is stationary then its expected amplitude throughout the duration of the signal pulse will be constant. The expected signal to RMS noise *voltage* ratio during pulse reception is therefore proportional to $v(t)$. (The expected signal to noise *power* ratio is proportional to $|v(t)|^2$.) It seems entirely reasonable, then, to weight each instantaneous value of signal plus noise voltage, $v_{in}(t)$, by the corresponding value of $v(t)$ and then to add (i.e. integrate) the result. (This corresponds to weighting each value of signal plus noise *power* by $|v(t)|^2$.)

If the reference signal, $v(t)$, is approximated by n sample values at regularly spaced time instants, $v(\Delta t), v(2\Delta t), \cdots, v((n-1)\Delta t), v(n\Delta t)$, the correlator can be implemented using a shift register, a set of weighting coefficients (the sample values) and an adder (Figure 8.34). This particular implementation has the form of a finite impulse response digital filter [Mulgrew and Grant] and illustrates, clearly, the equivalence of matched filtering and correlation detection.

A single matched filter, or correlation channel, is obviously adequate as a detector in the case of on-off keyed (OOK) systems since the output will be a maximum when a

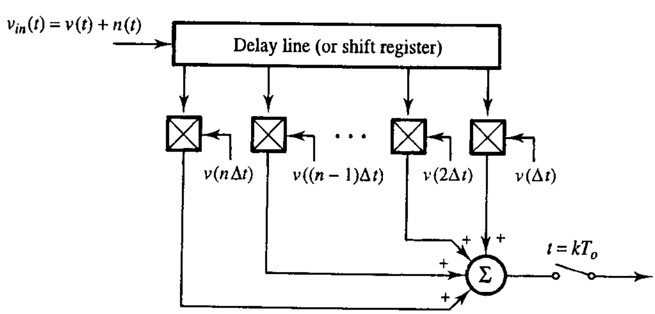


Figure 8.34 *Shift register implementation of matched filter illustrating relationship to correlation.*

pulse is present and essentially zero (ignoring noise) when no pulse is present. For binary systems employing two, non-zero, pulses a possible implementation would include two filters or correlators, one matched to each pulse type as shown in Figure 8.35. (This configuration will be used later for FSK detection, Chapter 11.) If the filter output is denoted by $f(t)$ then the possible sampling instant output voltages are:

$$f(kT_o) = \begin{cases} \int_0^{T_o} v_0^2(t)\,dt - \int_0^{T_o} v_0(t)v_1(t)\,dt, & \text{if symbol 0 is present} \\ -\int_0^{T_o} v_1^2(t)\,dt + \int_0^{T_o} v_1(t)\,v_0(t)\,dt, & \text{if symbol 1 is present} \end{cases} \tag{8.36}$$

If the signal pulses $v_0(t)$ and $v_1(t)$ are orthogonal but contain equal normalised energy, E_s V^2s (i.e. joules of energy dissipated in a 1 Ω load) then the sampling instant voltages will be $\pm E_s$. If the pulses are antipodal (i.e. $v_1(t) = -v_0(t)$) then the sampling instant voltages will be $\pm 2E_s$. The same output voltages can be generated, however, for all (orthogonal, antipodal or other) binary pulse systems using only one filter or correlator by matching to the pulse difference signal, $v_1(t) - v_0(t)$, as shown in Figure 8.36.

For multisymbol signalling the number of channels in the matched filter or correlation receiver can be extended in an obvious way, Figure 8.37. (If antipodal signal pairs are used in an M-ary system only $M/2$ detection channels are needed.)

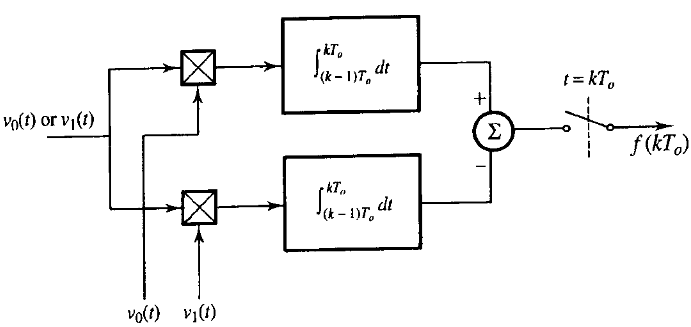


Figure 8.35 *Two channel, binary symbol correlator.*

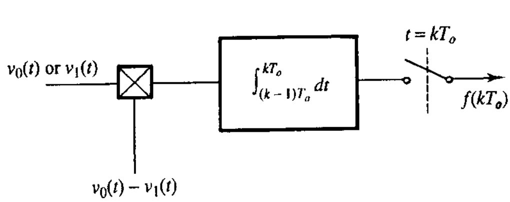


Figure 8.36 *One channel, binary symbol correlator.*

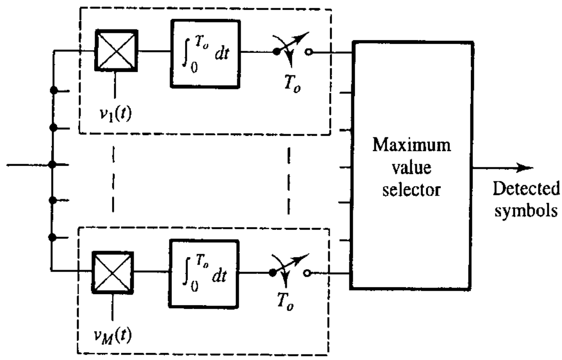


Figure 8.37 *Multichannel correlator for reception of M-ary signals.*

There is a disquieting aspect to equation (8.36) in that it seems dimensionally unsound. $f(kT_o)$ is a voltage yet the RHS of the equation has dimensions of normalised energy (V^2 s). This apparant paradox is resolved by considering the implementation of a correlator in more detail. The upper channel of the receiver in Figure 8.35, for example, contains a multiplier and an integrator. The multiplier must have an associated constant k_m with dimensions of V^{-1} if the output is to be a voltage (which it certainly is). Strictly, then, the output of the multiplier is $k_m v_0^2(t)$ volts when a digital 0 is present at its input. Similarly the integrator has a constant k_I which has dimensions of s^{-1}. (If this integrator is implemented as an operational amplifier, for example, with a resistor, R, in series with its inverting input and a capacitor, C, as its negative feedback element then $k_I = 1/(RC)$ (s^{-1}).) Since these constants affect signal voltages and noise voltages in identical ways they are usually ignored. This is equivalent to arbitrarily assigning to them a numerical value of 1.0 (resulting in an overall 'conversion' constant of 1.0 V/V^2s) and then (for orthogonal symbols) equating the numerical value of voltage at the correlator output with normalised symbol energy at the correlator input.

8.3.3 Decision instant SNR

A clue to the SNR performance of ideal matched filters and correlation detectors comes from equation (8.36). For orthogonal signal pulses the second (cross) terms in these equations are (by definition) zero. This leaves the first terms which represent the normalised energy, E_s, contained in the signal pulses. (The minus signs in equations (8.36) arise due to the subtractor placed after the integrators.) It is important to remember that it is the correlator output *voltage* which is numerically equal (assuming a 'conversion' constant of 1.0 V/V^2s) to the normalised symbol energy E_s, i.e.:

$$f(kT_o) = E_s \text{ (V)} \tag{8.37(a)}$$

The sampling instant normalised signal power at the correlator output is therefore:

$$|f(kT_o)|^2 = E_s^2 \text{ (V}^2\text{)} \tag{8.37(b)}$$

Since the noise at the correlator input is a random signal it must properly be described by its autocorrelation function (ACF) or its power spectral density. At the input to the multiplier the ACF of $n(t)$ is:

$$R_{nn}(\tau) = \langle n(t)\, n(t+\tau)\rangle \ (\text{V}^2) \tag{8.38}$$

Assuming that $n(t)$ is white with double sided power spectral density $N_0/2$ (V^2/Hz) then its ACF can be calculated by taking the inverse Fourier transform (Table 2.4) to obtain:

$$R_{nn}(\tau) = \frac{N_0}{2}\, \delta(\tau) \ (\text{V}^2) \tag{8.39}$$

The ACF of the noise after multiplication with $v(t)$ is:

$$\begin{aligned} R_{xx}(\tau) &= \langle x(t)\, x(t+\tau)\rangle \\ &= \langle n(t)v(t)\, n(t+\tau)v(t+\tau)\rangle \ (\text{V}^2) \end{aligned} \tag{8.40}$$

where $x(t) = n(t)v(t)$ and a 'multiplier constant' of 1.0 V/V^2 has been adopted. Since $n(t)$ and $v(t)$ are independent processes equation (8.40) can be rewritten as:

$$\begin{aligned} R_{xx}(\tau) &= \langle n(t)\, n(t+\tau)\rangle \langle v(t)\, v(t+\tau)\rangle \\ &= \frac{N_0}{2}\, \delta(\tau)\, R_{vv}(\tau) \ (\text{V}^2) \end{aligned} \tag{8.41}$$

$\delta(\tau)$ is zero everywhere except at $\tau = 0$, therefore:

$$R_{xx}(\tau) = \frac{N_0}{2}\, \delta(\tau)\, R_{vv}(0) \ \ (\text{V}^2) \tag{8.42}$$

$R_{vv}(0)$ is the mean square value of $v(t)$. Thus:

$$\begin{aligned} R_{xx}(\tau) &= \frac{N_0}{2}\, \delta(\tau)\, \frac{1}{T_o} \int_0^{T_o} v^2(t)\, dt \\ &= \frac{N_0}{2}\, \delta(\tau)\, \frac{E_s}{T_o} \ (\text{V}^2) \end{aligned} \tag{8.43}$$

Using the Wiener-Kintchine theorem (equation (3.48)) the two sided power spectral density of $x(t) = n(t)v(t)$ is the Fourier transform of equation (8.43), i.e.:

$$G_x(f) = \frac{N_0}{2}\, \frac{E_s}{T_o} \ (\text{V}^2/\text{Hz}) \tag{8.44}$$

The impulse response of a device which integrates from 0 to T_o seconds is a rectangle of unit height and T_o seconds duration (i. e. $\Pi[(t - T_o/2)/T_o]$). A good conceptual model of such a device is shown in Figure 8.38. The frequency response of this time windowed integrator (sometimes called a moving average filter) is the Fourier transform of its impulse response, i.e.:

$$H(f) = T_o\, \text{sinc}(T_o f)\, e^{-j\omega T_o/2} \tag{8.45}$$

and its amplitude response is therefore:

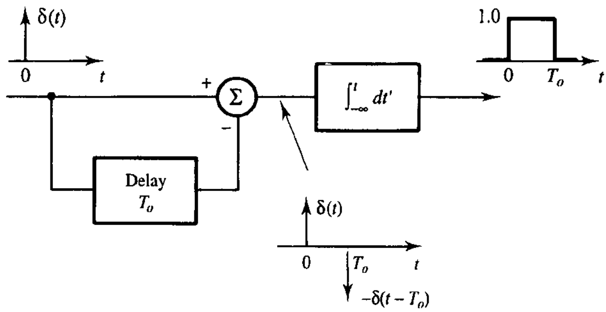


Figure 8.38 *Time windowed integrator (or moving average filter).*

$$|H(f)| = T_o\ |\text{sinc}(T_o f)| \tag{8.46}$$

The NPSD at the integrator output is:

$$G_y(f) = G_x(f)\ |H(f)|^2$$
$$= \frac{N_0}{2}\ E_s T_o\ \text{sinc}^2(T_o f) \tag{8.47}$$

and the total noise power at the correlator output is:

$$N = \frac{N_0}{2}\ E_s T_o \int_{-\infty}^{\infty} \text{sinc}^2(T_o f)\ df$$
$$= \frac{N_0}{2}\ E_s\ \ (\text{V}^2) \tag{8.48}$$

(The integral of the sinc^2 function can be seen by inspection to be $1/T_o$ using the Fourier transform 'value at the origin' theorem, Table 2.5.) The standard deviation of the noise at the correlator output (or, equivalently, its RMS value since its mean value is zero) is:

$$\sigma = \sqrt{N} = \sqrt{\left(\frac{N_0}{2}\ E_s\right)}\ \ (\text{V}) \tag{8.49}$$

Equations (8.37(a)) and (8.49) give a decision instant signal to RMS noise voltage ratio of:

$$\frac{f(T_o)}{\sigma} = \sqrt{\left(\frac{2E_s}{N_0}\right)} \tag{8.50}$$

or, alternatively, a decision instant signal to noise power ratio of:

$$\frac{S}{N} = \frac{|f(T_o)|^2}{\sigma^2} = \frac{2E_s}{N_0} \tag{8.51}$$

The important point here is that *the decision instant SNR at the output of a correlation* *receiver (or matched filter) depends only on pulse energy and input NPSD. It is independent of pulse shape.*

EXAMPLE 8.6
What is the sampling instant signal-to-noise ratio at the output of a filter matched to a triangular pulse of height 10 mV and width 1.0 ms if the noise at the input to the filter is white with a power spectral density of 10 nV^2/Hz?

Energy in the input pulse is given by:

$$E_s = \int_0^{T_o} v^2(t)\, dt = \int_0^{\frac{T_o}{2}} v^2(t)\, dt + \int_{\frac{T_o}{2}}^{T_o} v^2(t)\, dt$$

$$= \int_0^{0.5\times 10^{-3}} [20\, t]^2\, dt + \int_{0.5\times 10^{-3}}^{1\times 10^{-3}} [2\times 10^{-2} - 20\, t]^2\, dt$$

$$= 400\left[\frac{t^3}{3}\right]_0^{0.5\times 10^{-3}} + 4\times 10^{-4}\left[\frac{t}{1}\right]_{0.5\times 10^{-3}}^{1\times 10^{-3}}$$

$$- 80\times 10^{-2}\left[\frac{t^2}{2}\right]_{0.5\times 10^{-3}}^{1\times 10^{-3}} + 400\left[\frac{t^3}{3}\right]_{0.5\times 10^{-3}}^{1\times 10^{-3}}$$

$$= 0.33\times 10^{-7}\ (V^2\ s)$$

Using equation (8.51) the sampling instant SNR when a pulse is present at the filter input is:

$$\frac{S}{N} = \frac{2E_s}{N_0} = \frac{2\times 0.33\times 10^{-7}}{10\times 10^{-9}} = 6.67 = 8.2\ \text{dB}$$

8.3.4 BER performance of optimum receivers

A general formula giving the probability of symbol error for an optimum binary receiver (matched filter or correlator) is most easily derived by considering a single channel correlator matched to the binary symbol difference, $v_1(t) - v_0(t)$. When a binary 1 is present at the receiver input the decision instant voltage at the output is given by equation (8.36) as:

$$f(kT_o) = \int_{(k-1)T_o}^{kT_o} v_1(t)\, [v_1(t) - v_0(t)]\, dt$$

$$= E_{s1} - \int_{(k-1)T_o}^{kT_o} v_1(t)v_0(t)\,dt \tag{8.52}$$

where E_{s1} is the binary 1 symbol energy. When a binary 0 is present at the receiver input the decision instant voltage is:

$$f(kT_o) = -E_{s0} + \int_{(k-1)T_o}^{kT_o} v_1(t)v_0(t)\,dt \tag{8.53}$$

where E_{s0} is the binary 0 symbol energy. The second term of both equations (8.52) and (8.53) represents the correlation between symbols. Defining the normalised correlation coefficient to be:

$$\rho = \frac{1}{\sqrt{(E_{s0}\,E_{s1})}} \int_{(k-1)T_o}^{kT_o} v_1(t)\,v_0(t)\,dt \tag{8.54}$$

then equations (8.52) and (8.53) can be written as:

$$f(kT_o) = \begin{cases} E_{s1} - \rho\sqrt{(E_{s0}E_{s1})}, & \text{for binary 1} \\ -E_{s0} + \rho\sqrt{(E_{s0}E_{s1})}, & \text{for binary 0} \end{cases} \tag{8.55}$$

(The proper interpretation of equation (8.54) when $v_0(t) = 0$ and therefore $E_{s0} = 0$ (i.e. OOK signalling, section 6.4.1) is $\rho = 0$.) The difference in decision instant voltages representing binary 1 and 0 is:

$$\Delta V = E_{s1} + E_{s0} - 2\rho\sqrt{(E_{s1}\,E_{s0})} \tag{8.56}$$

Equation (8.56) also represents the energy, E_s', in the reference pulse of the single channel correlator, i.e.:

$$E_s' = \int_0^{T_o} [v_1(t) - v_0(t)]^2\,dt = \Delta V \tag{8.57}$$

The RMS noise voltage at the output of the receiver is given by equation (8.49), i.e.:

$$\sigma = \sqrt{\left(\frac{N_0}{2}\,E_s'\right)}$$

$$= \left[\frac{N_0}{2}\left(E_{s1} + E_{s0} - 2\rho\sqrt{(E_{s1}\,E_{s0})}\right)\right]^{1/2} \tag{8.58}$$

The quantity $\Delta V/\sigma$ is therefore:

$$\frac{\Delta V}{\sigma} = \left[\frac{2}{N_0}\left(E_{s1} + E_{s0} - 2\rho\sqrt{(E_{s1}\,E_{s0})}\right)\right]^{1/2} \tag{8.59}$$

For binary symbols of equal energy, E_s, this simplifies to:

$$\frac{\Delta V}{\sigma} = 2\sqrt{\left[\frac{E_s}{N_0}(1-\rho)\right]} \tag{8.60}$$

Equation (8.59) or (8.60) can be substituted into the centre point sampling formula (equation (6.8)) to give the ideal correlator (or matched filter) probability of symbol error. In the (usual) equal symbol energy case this gives:

$$P_e = \frac{1}{2}\left\{1 - \text{erf}\sqrt{\left[\frac{E_s}{2N_0}(1-\rho)\right]}\right\} \tag{8.61}$$

Although strictly speaking equations (8.60) and (8.61) are valid only for equal energy binary symbols they also give the correct probability of error for OOK signalling *providing* that E_s is interpreted as the average energy per symbol (i.e. half the energy of the non-null symbol). For all orthogonal signalling schemes (including OOK) $\rho = 0$. For all antipodal schemes (in which $v_1(t) = -v_0(t)$) $\rho = -1$.

EXAMPLE 8.7

A baseband binary communications system transmits a positive rectangular pulse for digital ones and a negative triangular pulse for digital zeros. If the (absolute) widths, peak pulse voltages, and noise power spectral density at the input of an ideal correlation receiver are all identical to those in Example 8.6 find the probability of bit error.

The energy in the triangular pulse has already been calculated in Example 8.6:

$$E_{s0} = 0.33 \times 10^{-7} \text{ V}^2\text{s}$$

The energy in the rectangular pulse is:

$$E_{s1} = v^2 T_o = (10 \times 10^{-3})^2 \times 1 \times 10^{-3} = 1 \times 10^{-7} \text{ V}^2 \text{ s}$$

Using equation (8.54):

$$\rho = \frac{1}{\sqrt{E_{s0} E_{s1}}} \int_0^{T_o} v_1(t)\, v_0(t)\, dt$$

$$= \frac{1}{\sqrt{0.33 \times 10^{-7} \times 1 \times 10^{-7}}} \int_0^{10^{-3}} \left[-10 \times 10^{-3}\, \Pi\left(\frac{t - 0.5 \times 10^{-3}}{10^{-3}}\right)\right.$$

$$\left. \times\, 10 \times 10^{-3}\, \Lambda\left(\frac{t - 0.5 \times 10^{-3}}{5 \times 10^{-4}}\right)\right] dt$$

$$= -1.74 \times 10^3 \times 2\left[2 \times 10^3\, t^2/2\right]_0^{0.5 \times 10^{-3}} = -0.87$$

Using equation (8.59):

$$\frac{\Delta V}{\sigma} = \left[\frac{2}{N_0} \left(E_{s1} + E_{s0} - 2\,\rho\,\sqrt{(E_{s1}\,E_{s0})} \right) \right]^{½}$$

$$= \left[\frac{2}{10 \times 10^{-9}} (0.33 \times 10^{-7} + 1.0 \times 10^{-7} - 2\,(-0.87)\,\sqrt{0.33 \times 10^{-14}}) \right]^{½}$$

$$= 6.83$$

Using equation (6.8):

$$P_e = \frac{1}{2}\left[1 - \text{erf}\,\frac{\Delta V}{2\sqrt{2}\,\sigma}\right]$$

$$= \frac{1}{2}\left[1 - \text{erf}\,\frac{6.83}{2\sqrt{2}}\right] = 3.2 \times 10^{-4}$$

8.3.5 Comparison of baseband matched filtering and centre point detection

Equation (8.59) can be used to compare the performance of a baseband matched filter receiver with simple centre point detection of rectangular pulses as discussed in Chapter 6. For unipolar NRZ transmission equation (8.59) shows that the detection instant $\Delta V/\sigma$ after matched filtering is related to that for simple centre point detection (CPD) by:

$$\left(\frac{\Delta V}{\sigma}\right)_{MF} = \left(\frac{2\,E_{s1}}{N_0}\right)^{½}$$

$$= \left(\frac{2\,\Delta V^2 T_o}{\sigma^2/B}\right)^{½}$$

$$= \sqrt{2}(T_o B)^{½}\left(\frac{\Delta V}{\sigma}\right)_{CPD} \qquad (8.62)$$

where T_o is the rectangular pulse duration and B is the CPD predetection (rectangular) bandwidth. It may be disturbing to recognise that if rectangular pulse CPD transmission is interpreted literally then B must be infinite to accommodate infinitely fast rise and fall times. However, in this (literal) case $(\Delta V/\sigma)_{CPD}$ is zero due to the infinite noise power implied by a white noise spectrum. In practice, the CPD predetection bandwidth B is limited to a finite value (say 2 or 3 times $1/T_o$) and T_o is interpreted as the symbol period to allow the resulting spreading of the symbol in time. The saving of transmitter power (or allowable increase in noise power spectral density) that matched filtering provides for compared with CPD is therefore:

$$\frac{(\Delta V/\sigma)_{MF}}{(\Delta V/\sigma)_{CPD}} = \sqrt{2}(T_o B)^{½}$$

$$= 3.0 + (T_o B)_{dB} \quad \text{(dB)} \tag{8.63}$$

A CPD predetection bandwidth of three times the baud rate ($B = 3/T_o$), for example, therefore gives a power saving of 7.8 dB. (Although equation (8.63) has been derived for unipolar transmission it is also correct for the polar transmission case.)

For $B = 0.5/T_o$ (the minimum bandwidth consistent with ISI free transmission), then the performance of matched filtering and CPD is the same. This apparent paradox is resolved by appreciating that this minimum bandwidth implies sinc pulse transmission in which case the rectangular predetection CPD filter is itself precisely matched to the expected symbol pulse shape.

8.3.6 Differences between matched filtering and correlation

Although matched filters and correlation detectors give identical detection instant signal and noise voltages at their outputs for identical inputs (and therefore have identical P_e performance) they do not *necessarily* give the same pulse shapes at their outputs. This is because in the case of the correlator (Figure 8.33) the received pulse and reference pulse are aligned in time throughout the pulse duration whereas in the case of the filter (Figure 8.34) the received pulse slides across the reference pulse giving the true ACF (neglecting noise) of the input pulse. Specifically the pulse at the output of the correlator is given by:

$$f(t) = \int_0^t v^2(t')\, dt' \tag{8.64}$$

whilst the pulse at the output of the matched filter (see equation (8.33)) is given by:

$$f(t) = \int_0^t v(t')\, v(t' + T_o - t)\, dt' \tag{8.65}$$

(The lower limits in the integrals of equations (8.64) and (8.65) assume that the pulses start at $t = 0$.) This difference has no influence on P_e providing there are no errors in symbol timing. If, however, decision instants are not perfectly timed then there is the possibility of a discrepancy in matched filter and correlator performance. This is well illustrated by the case of rectangular RF pulse signalling. Figure 8.39 shows the detector output pulses for a matched filter and a correlator. It is clear that, providing the timing instant never occurs after $t = T_o$, the matched filter would suffer greater performance degradation due to symbol timing errors, for this type of pulse, than the correlator.

8.4 Root raised cosine filtering

Nyquist filtering and matched filtering have both been identified as optimum filtering techniques, the former because it results in ISI free signalling in a bandlimited channel and the latter because it results in maximum SNR at the receiver decision instants. Whilst Nyquist filtering was discussed in the context of transmitter filters it is important

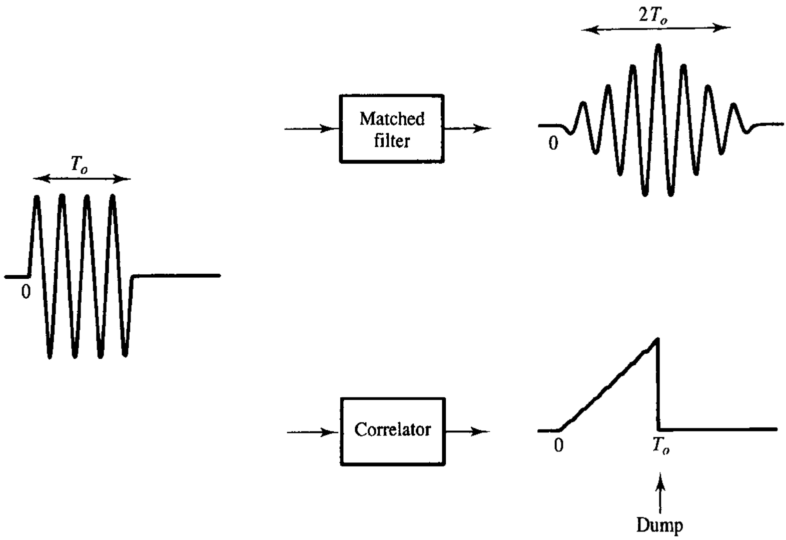


Figure 8.39 *Output pulses of matched filter and correlator for a rectangular RF input pulse.*

to realise that the Nyquist frequency response which gives ISI free detection includes the pulse shaping filter at the transmitter, the frequency response of the transmission medium and any filtering in the receiver prior to the decision circuit, i.e.:

$$H_N(f) = H_T(f)\, H_{ch}(f)\, H_R(f) \tag{8.66}$$

where the subscripts denote the Nyquist, transmitter, channel and receiver frequency responses respectively. Assuming that the channel introduces negligible distortion ($H_{ch}(f) = 1$) then it is clear that the Nyquist frequency response can be split in any convenient way between the transmitter and receiver. It is also clear that if the transmitter and receiver filter are related by:

$$H_T^*(f) = H_R(f) \tag{8.67}$$

then the spectrum of the transmitted pulses (assuming impulses prior to filtering) will be the conjugate of the frequency response of the receiver. Apart from a linear phase factor this is precisely the requirement for matched filtering. It is therefore possible, by judicious splitting of the overall system frequency response, to satisfy both the Nyquist and matched filtering criteria simultaneously. A popular choice for $H_T(f)$ and $H_R(f)$ is the root raised cosine filter (Figure 8.40(a)) derived from equation (8.8), i.e.:

$$\begin{aligned} H_T(f) &= H_R(f) \\ &= \begin{cases} \sqrt{\cos^2(\pi f/4f_\chi)}, & f \le 2f_\chi \\ 0, & f > 2f_\chi \end{cases} \end{aligned} \tag{8.68}$$

where $f_\chi = 1/(2T_o)$. The overall frequency response then has a full raised cosine

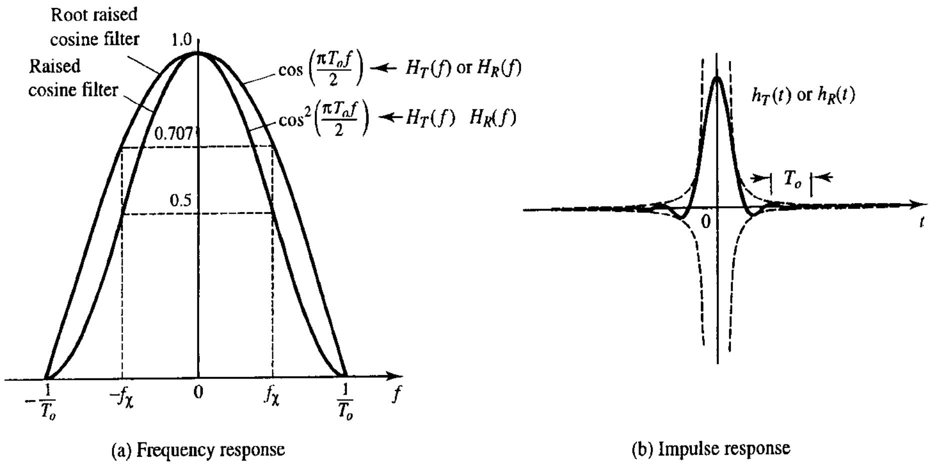


Figure 8.40 *Root raised cosine filter responses.*

characteristic giving ISI free detection. The impulse response of the root raised cosine filter, given by the inverse Fourier transform of equation (8.68), is:

$$h(t) = \frac{8f_\chi}{\pi} \frac{\cos(4\pi f_\chi t)}{1 - 64f_\chi^2 t^2} \tag{8.69}$$

This impulse response, Figure 8.40(b), is, of course, the transmitted pulse shape.

The similarity between the root raised cosine filter and the cosine filter used in duobinary signalling is obvious. The difference is in their bandwidth. The bandwidth B, of the root raised cosine filter is $B = 1/T_o$ Hz. The bandwidth of the (duobinary) cosine filter is $B = 1/(2T_o)$ Hz.

8.5 Equalisation

When a digital signal is transmitted over a realistic channel it can be severely distorted. The communications channel including transmitter filters, multipath effects and receiver filters can be modelled by a finite impulse response (FIR) filter, with the same structure as that shown in Figure 8.34 [Mulgrew and Grant]. The transmitted data can often be effectively modelled as a discrete random binary sequence, $x(kT_o)$, which can take on values of, say, ±1 V. Gaussian noise samples, $n(kT_o)$, are added to the FIR filter (i.e. channel) output resulting in the received samples, $f(kT_o)$. In the simplest case all the coefficients of the FIR filter would be zero except for one tap which would have weight, h_0. The received signal samples would then be:

$$f(kT_o) = \pm h_0 + n(kT_o) \tag{8.70}$$

and we could tell what data was being transmitted by simply testing whether $f(kT_o)$ is greater than or less than zero, i.e.:

$$\text{if } f(kT_o) \geq 0 \quad \text{then } x(kT_o) = +1 \quad \text{else } x(kT_o) = -1 \tag{8.71}$$

The received sample $f(kT_o)$ is more often, however, a function of several transmitted bits or symbols as defined by the number of taps with significant weights. This merging of samples represents intersymbol interference, as discussed in section 8.2.

Each channel has a particular frequency response. If we knew this frequency response we could include a filter in the receiver with the 'opposite' or inverse frequency response, as discussed in section 6.6.2. Everywhere the channel had a peak in its frequency response the inverse filter would have a trough and vice versa. The frequency response of the channel in cascade with the inverse filter would ideally, have a, wideband, flat amplitude response and a linear phase response, i.e. as far as the transmitted signal was concerned the cascade of the channel and the inverse filter together would look like a simple delay. Effectively we would have equalised the frequency response of the channel.

Note that the equaliser operation is fundamentally wideband, compared to the matched filter of Figure 8.34. When a signal is corrupted by white noise the matched filter detector possesses a frequency response which is *matched accurately* to the expected signal characteristic. Consequently the matched filter bandwidth equals the signal bandwidth. The equalising filter bandwidth is typically much greater than the signal bandwidth, however, to achieve a commensurate narrower duration output pulse response, than with the matched filter operation.

In practice when we switch on our digital mobile radio, or telephone modem, we have no idea what the frequency response of the channel between the transmitter and the receiver will be. In this type of application the receiver equaliser must, therefore, be adaptive. If such equalisers are to approximate the optimal filter or estimator they require explicit knowledge of the signal environment in the form of correlation functions, power delay profiles, etc. In most situations such functions are unknown and/or time-varying. The equaliser must therefore employ a closed loop (feedback) arrangement in which its frequency response is adapted, or controlled, by a feedback algorithm. This permits it to compensate for time-varying distortions and still achieve performance, close to the optimal estimator function.

Adaptive filters [Mulgrew and Grant] use an adjustable or programmable filter whose impulse response is controlled to pass the desired components of each signal sample and to attenuate the undesired components in order to compensate distortion present in the input signal. This may be achieved by employing a known data sequence or training signal, Figure 8.41. An input data plus noise, sample sequence, $f(kT_o)$, is convolved with a time-varying FIR sequence, $h_i(kT_o)$. The output of the N-tap filter is $\hat{x}(kT_o)$, is given by the discrete convolution operation (see section 13.6):

$$\hat{x}(kT_o) = \sum_{i=0}^{N-1} h_i(kT_o) f((k-i)T_o) \tag{8.72}$$

The filter output, $\hat{x}(kT_o)$, is used as the estimate of the training signal, $x(kT_o)$, and is subtracted from this signal to yield an error signal:

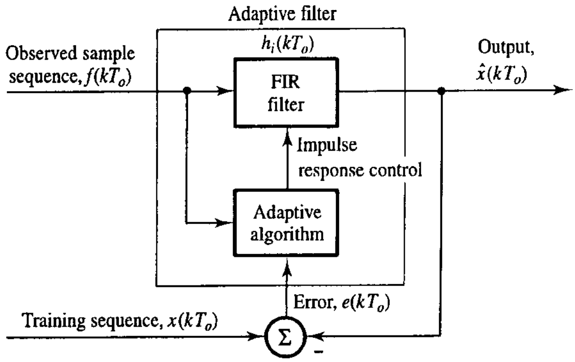


Figure 8.41 *Adaptive filter operation.*

$$e(kT_o) = x(kT_o) - \hat{x}(kT_o) \tag{8.73}$$

The error is then used in conjunction with the input signal, $f(kT_o)$, to determine the next set of filter weight values, $h_i((k+1)T_o)$.

The impulse response of the adaptive filter, $h_i(kT_o)$, is thus progressively altered as more of the observed, and training, sequences become available, such that the output $\hat{x}(kT_o)$ converges to the training sequence $x(kT_o)$ and hence the output of the optimal filter. Adaptive filters again employ a finite impulse response structure, as this is more stable than other filter forms, such as recursive infinite impulse response designs.

Figure 8.42 illustrates how this technique can be applied for practical data communications. Here the input, $f(kT_o)$, is genuine data, $x(kT_o)$, convolved with the communication channel impulse response, plus additive noise. When the transmitter is switched on, however, it sends a training sequence prior to the data. The objective here is to make the output, $\hat{x}(kT_o)$, approximate the training sequence and in so doing 'teach' the adaptive algorithm the required impulse response of the inverse filter.

The adaptive algorithm can then be switched off and genuine data transmitted. On conventional telephone lines the channel response does not change with time once the circuit has been established. Having trained the equaliser the adaptive algorithm can,

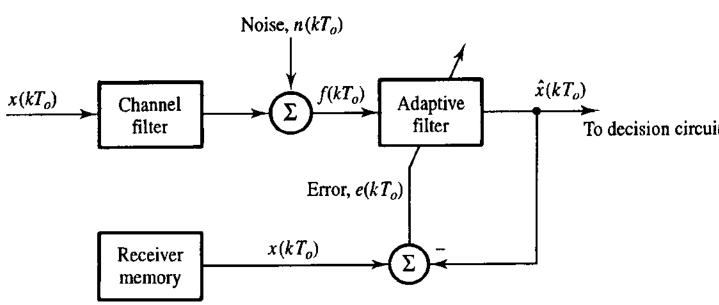


Figure 8.42 *Use of adaptive filter for adaptive equalisation.*

therefore, be disconnected until a new line is dialled up.

In digital mobile radio applications, for example, this is not the case since the channel between the transmitter and receiver will change with time, and also with the position and velocity of the mobile transceiver. Thus the optimum filter is required to track changes in the channel.

One way of tackling this problem is to use what is called 'decision directed' operation, Figure 8.43. If the equaliser is working well then $\hat{x}(kT_o)$ will just be the binary data plus noise. We can remove the noise in a decision circuit which simply tests whether $\hat{x}(kT_o)$ is greater or less than zero. The output of the decision circuit is then identical to the transmitted data. We can use this data to continue training the adaptive filter by changing the switch in Figure 8.43 to its lower position. Here the error is the difference between the binary data and the filter output. This 'decision directed' system will work well provided the receiver continues to make correct decisions about the transmitted data. The decision directed equaliser thus operates effectively in slowly changing channels where the adaptive filter feedback loop can track these changes. If the filter cannot track the signal then the switch in Figure 8.43 must be moved to the upper position and the training sequence retransmitted to initialise the filter, as previously in Figure 8.42. The adaptive filter concept is fundamental to the operation of digital cellular systems which must overcome the channel fading effects described in section 15.2.

A useful way of quickly assessing the performance of a digital communications equaliser is to display its output as an eye diagram on an oscilloscope as described in section 6.6.3.

8.6 Summary

Two types of optimum filtering are important to digital communications. Nyquist filtering constrains the bandwidth of a signal whilst avoiding sampling instant ISI at the

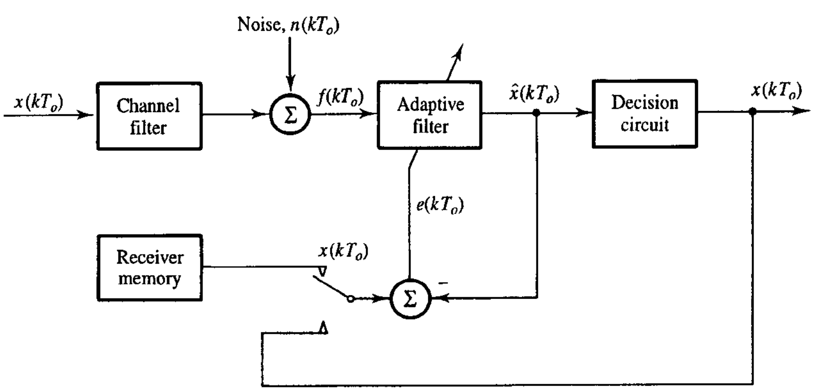


Figure 8.43 *Decision directed adaptive equaliser operation.*

decision circuit input. Matched filtering maximises the sampling instant SNR at the decision circuit input. Both types of filtering are optimum, therefore, in the sense that they minimise the probability of bit error. Nyquist filtering can be implemented entirely at the transmitter if no distortion occurs in the transmission channel or receiver. Implementation is often, however, distributed between transmitter and receiver, distortion in the channel being cancelled by a separate equaliser. The amplitude response of a Nyquist filter has odd symmetry about its –6 dB frequency points and its phase response is linear.

Duobinary signalling uses pulses, transmitted at a baud rate of $1/(2B)$ symbol/s, which suffer from severe, but predictable, ISI. The predictability of the ISI means that it can be cancelled by appropriate signal, or symbol, processing at the receiver, thus effectively allowing ISI free transmission at the maximum theoretical baud rate. Partial response signalling represents a generalisation of the duobinary technique to multilevel signalling. In this case the (predictable) ISI extends across a window of several adjacent symbols.

Matched filtering is implemented at the receiver. The amplitude response of this filter is proportional to the amplitude spectrum of the symbol to which it is matched and its phase response is, to within a linear phase factor, opposite to the phase spectrum of the symbol. Correlation detection is matched filtering implemented in the time domain.

Root raised cosine filters (applied to impulse signalling at the transmitter and the transmitted symbols at the receiver) satisfy both Nyquist and matched filtering criteria, assuming a distortionless, or perfectly equalised, channel. Channels which are time varying may require adaptive equalisation.

8.7 Problems

8.1. A binary information source consists of statistically independent, equiprobable, symbols. If the bandwidth of the baseband channel over which the symbols are to be transmitted is 3.0 kHz what baud rate will be necessary to achieve a spectral efficiency of 2.5 bit/s/Hz? Is ISI free reception at this baud rate possible? What must be the minimum size of the source symbol alphabet to achieve ISI free reception and a spectral efficiency of 16 bit/s/Hz? [7.5 kbaud, no, 256]

8.2. What is the Nyquist filtering criterion expressed in: (a) the time domain; and (b) the frequency domain?

8.3. State Nyquist's vestigial symmetry theorem. Why is this theorem useful in the context of digital communications?

8.4. Which is the more general, the family of Nyquist filters or the family of raised cosine filters? Sketch the amplitude response of: (a) a baseband raised cosine filter with a normalised excess bandwidth of 0.3; and (b) a bandpass full raised cosine filter.

8.5. Given that a Nyquist filter has odd symmetry about its parent rectangular filter's cut-off frequency, demonstrate that the impulse response of the Nyquist filter retains those zeros present in the impulse response of the top hat filter. [Hint: Consider how the odd symmetry of the Nyquist filter's frequency response could be obtained by convolving the rectangular function with an even function.]

8.6 Justify Nyquist's vestigial symmetry theorem, as encapsulated in Figure 8.8, directly (i.e.

without recourse to the argument used in Problem 8.5).

8.7. A baseband binary (2-level) PCM system is used to transmit a single 3.4 kHz voice signal. If the sampling rate for the voice signal is 8 kHz and 256 level quantisation is used, calculate the bandwidth required. Assume that the total system frequency response has a full raised cosine characteristic. [64 kHz]

8.8. An engineer proposes to use sinc pulse signalling for a baseband digital communications system on the grounds that the sinc pulse is a special case of a Nyquist signal. Briefly state whether you support or oppose the proposal and, on what grounds, your case is based.

8.9. A baseband transmission channel has a raised-cosine frequency response with a roll-off factor, $\alpha = 0.4$. The channel has an (absolute) bandwidth of 1200 kHz. An analogue signal is converted to binary PCM with 64-level quantisation before being transmitted over the channel. What is the maximum limit on the bandwidth of the analogue signal? What is the maximum possible spectral efficiency of this system? [143 kHz, 1.43 bit/s/Hz]

8.10. A voice signal is restricted to a bandwidth of 3.0 kHz by an ideal anti-aliasing filter. If the bandlimited signal is then over-sampled by 33% find the ISI free bandwidth required for transmission over a channel, having a Nyquist response with a roll-off factor of 50%, for the following schemes: (a) PAM; (b) 8-level quantisation, binary PCM; and (c) 64-level quantisation, binary PCM. [6 kHz, 18 kHz, 36 kHz]

8.11. A 4-level PCM communications system has a bit rate of 4.8 kbit/s and a raised cosine total system frequency response with a roll-off factor of 0.3. What is the minimum transmission bandwidth required? [N.B. A 4-level PCM system is one in which information signal samples are coded into a 4-level symbol stream rather than the usual binary symbol stream.] [1.56 kHz]

8.12. Demonstrate that duobinary signalling suffers from error propagation whilst precoded duobinary signalling does not.

8.13. (a) What is the principal objective of matched filtering? (b) What is the (white noise) matched filtering criterion expressed in: (i) the time domain; and (ii) the frequency domain?
(c) How is the sampling instant SNR at the output of a matched filter related to the energy of the expected symbol and NPSD at the filter's input?

8.14. Sketch the impulse response of the filter which is matched to the pulse:

$$f(t) = \Pi(t - 0.5) + (2/3)\,\Pi(t - 1.5)$$

What is the output of this filter when the pulse $f(t)$ is present at its input?

8.15. The transmitted pulse shape of an OOK, baseband, communication system is $1000t\Pi([t - 0.5 \times 10^{-3}]/10^{-3})$. What is the impulse response of the predetection filter which maximises the sampling instant, SNR (i.e. the matched filter)? If the noise at the input to the predetection filter is white and Gaussian with a one-sided power spectral density of 2.0×10^{-5} V^2/Hz, what probability of symbol error would you expect in the absence of intersymbol interference? [2×10^{-4}]

8.16. A polar binary signal consists of +1 or −1 V pulses during the interval (0, T). Additive white Gaussian noise having a two sided NPSD of 10^{-6} W/Hz is added to the signal. If the received signal is detected with a matched filter, determine the maximum bit rate that can be sent with an error probability, P_e, of less than or equal to 10^{-3}. Assume that the impedance level is 50 ohms.
What is the sampling instant SNR in dB at the output of the filter?
Can you say anything about the SNR at the filter input? [2.1 kbit/s, 9.8 dB]

8.17. The time-domain implementation of a matched filter is called a *correlation* detector. In view of the fact that the output of a filter is the *convolution* of its input with its impulse response explain why this terminology is appropriate.

8.18. A binary baseband communications system employs the transmitted pulses $v_0(t) = -\Lambda(t/(0.5 \times 10^{-3}))$ V and $v_1(t) = \Pi(t/10^{-3})$ V to represent digital zeros and ones respectively. What is the ideal, single channel, correlator reference signal for this system? If the loss from transmitter to receiver is 40.0 dB what value of noise power spectral density at the correlator input can be tolerated whilst maintaining a probability of symbol error of 10^{-6}? [5.15×10^{-9} V^2/Hz]

8.19. A digital communications receiver uses root raised cosine filtering at both its transmitter and receiver. Show that the transmitted pulse has the form:

$$v(t) = \frac{4}{\pi} R_s \frac{\cos(2\pi R_s t)}{1 - 16R_s^2 t^2}$$

where R_s is the baud rate.