CHAPTER 6

Baseband transmission and line coding

6.1 Introduction

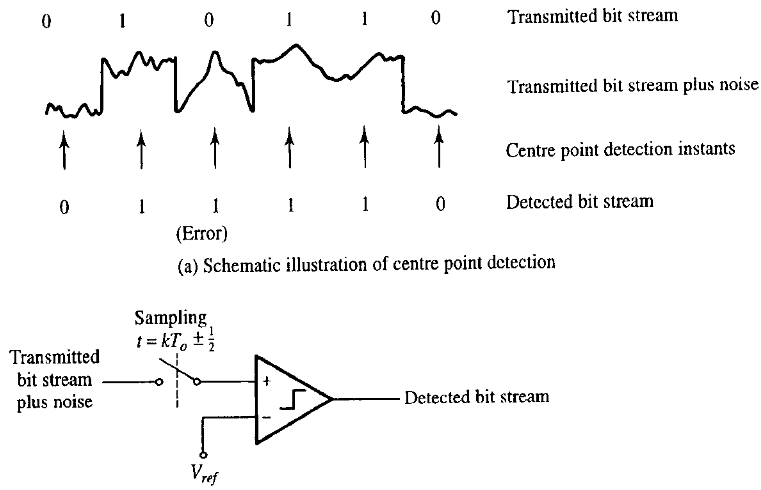
This chapter addresses two issues central to baseband signal transmission, digital signal detection and line coding. The detection process described here is restricted to a simple implementation of the decision circuit in Figure 1.3. The analysis of detection processes is important since this leads to the principal objective measure of digital communication systems quality, i.e. the probability of symbols being detected in error, as quantified by probability theory (section 3.2). Appropriate coding of the transmitted symbol pulse shape can minimise the probability of error by ensuring that the spectral characteristics of the digital signal are well matched to the transmission channel. The line code must also permit the receiver to extract accurate PCM bit and word timing signals directly from the received data, for proper detection and interpretation of the digital pulses.

6.2 Baseband centre point detection

The detection of digital signals involves two processes:

1. Reduction of each received voltage pulse (i.e. symbol) to a single numerical value.
2. Comparison of this value with a reference voltage (or, for multisymbol signalling, a set of reference voltages) to determine which symbol was transmitted.

In the case of symbols represented by different voltage levels the simplest way of achieving 1 is to sample the received signal plus noise; 2 is then implemented using one or more comparators. In the case of equiprobable, binary, symbols (zero and one) represented by two voltage levels (e.g. 0 V and 3 V) intuition tells us that a sensible strategy would be to set the reference, V_{ref}, (Figure 6.1(b)) mid-way between the two voltage levels (i.e. at 1.5 V). Decisions would then be made at the centre of each symbol period on the basis of whether the instantaneous voltage (signal plus noise) is above or below this reference. Sampling the instantaneous signal plus noise voltage somewhere near the middle of the symbol period is called *centre point detection*, Figure 6.1(a).



(a) Schematic illustration of centre point detection

(b) Principle of centre point detector

Figure 6.1 *Centre point detection.*

The noise present during detection is often either Gaussian or approximately Gaussian. (This is always the case if thermal noise is dominant, but may also be the case when other sources of noise dominate owing to operation of the central limit theorem.) Since noise with a Gaussian probability density function (pdf) is common and analytically tractable, the bit error rate (BER) of a communications system is often modelled assuming this type of noise alone.

6.2.1 Baseband binary error rates in Gaussian noise

Figure 6.2(a) shows the pdf of a binary information signal which can take on voltage levels V_0 and V_1 only. Figure 6.2(b) shows the probability density function of a zero mean Gaussian noise process, $v_n(t)$, with RMS value σ V. (Since the process has zero mean the RMS value and standard deviation are identical.) Figure 6.2(c) shows the pdf of the sum of the signal and noise voltages. (Whilst Figure 6.2(c) is perhaps no surprise – we can think of the 'quasi-DC' symbol voltages biasing the mean value of the noise to V_0 and V_1 – recall, from Chapter 3, that when independent random variables are added their pdfs are convolved. Figure 6.2(c) is thus the convolution of Figures 6.2(a) and 6.2(b).)

For equiprobable symbols the optimum decision level is set at $(V_0 + V_1)/2$. (This is not the optimum threshold if the symbols are not equiprobable: see section 7.4.4.) Given that the symbol 0 is transmitted (i.e. a voltage level V_0) then the probability, P_{e1}, that the signal plus noise will be above the threshold at the decision instant is given by twice [1] the

[1] $p_0(v_n)$ and $p_1(v_n)$, as defined here, each represent a total probability of 0.5. Strictly speaking they are not, therefore, pdfs although their sum is the total signal plus noise pdf irrespective of whether a one or zero is transmitted. The pdf of the signal plus noise conditional on a zero being transmitted is $2p_0(v_n)$.

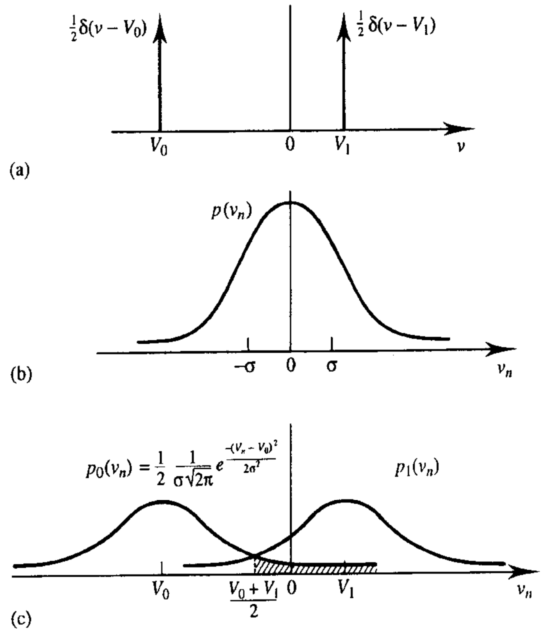


Figure 6.2 *Probability density function of: (a) binary symbol; (b) noise; (c) signal plus noise.*

shaded area under the curve $p_0(v_n)$ in Figure 6.2(c), i.e.:

$$P_{e1} = \int_{(V_0+V_1)/2}^{\infty} \frac{1}{\sigma\sqrt{(2\pi)}} e^{\frac{-(v_n-V_0)^2}{2\sigma^2}} dv_n \tag{6.1}$$

Using the change of variable $x = (v_n - V_0)/\sqrt{2}\sigma$ this becomes:

$$P_{e1} = \frac{1}{\sqrt{\pi}} \int_{(V_1-V_0)/2\sqrt{2}\sigma}^{\infty} e^{-x^2}\, dx \tag{6.2}$$

The incomplete integral in equation (6.2) cannot be evaluated analytically but can be recast as a complementary error function, erfc(x), defined by:

$$\text{erfc}(z) \underline{\Delta} \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-x^2}\, dx \tag{6.3}$$

Thus equation (6.2) becomes:

$$P_{e1} = \frac{1}{2}\, \text{erfc}\left(\frac{V_1 - V_0}{2\sigma\sqrt{2}}\right) \tag{6.4}$$

Alternatively, since erfc(z) and the error function, erf(z), are related by:

$$\text{erfc}(z) \equiv 1 - \text{erf}(z) \tag{6.5}$$

then P_{e1} can also be written as:

$$P_{e1} = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V_1 - V_0}{2\sigma\sqrt{2}}\right)\right] \tag{6.6}$$

The advantage of using the error (or complementary error) function in the expression for P_{e1} is that this function has been extensively tabulated[2] (see Appendix A).

If the digital symbol one is transmitted (i.e. a voltage level V_1) then the probability, P_{e0}, that the signal plus noise will be below the threshold at the decision instant is:

$$P_{e0} = \int_{-\infty}^{(V_0+V_1)/2} \frac{1}{\sigma\sqrt{(2\pi)}} e^{\frac{-(v_n-V_1)^2}{2\sigma^2}} dv_n \tag{6.7}$$

It is clear from the symmetry of this problem that P_{e0} is identical to both P_{e1} and the probability of error, P_e, irrespective of whether a one or zero was transmitted. Noting that the probability of error depends on only symbol voltage difference, and not absolute voltage levels, P_e can be rewritten in terms of $\Delta V = V_1 - V_0$, i.e.:

$$P_e = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right] \tag{6.8}$$

Equation (6.8) is valid for both *unipolar* signalling (i.e. symbols represented by voltages of 0 and ΔV) and *polar signalling* (i.e. symbols represented by voltages of $\pm\Delta V/2$). In fact, it is valid for all pulse levels and shapes providing that ΔV represents the voltage *difference at the sampling instant.* Figure 6.3 shows P_e versus the voltage ratio $\Delta V/\sigma$ expressed both in dB and as a ratio.

Whilst the x-axis of Figure 6.3 is clearly related to signal-to-noise ratio it is not identical to it for all pulse levels and shapes. This is partly because it involves the signal only at the sampling instant (whereas a conventional SNR uses a time averaged signal power) and partly because the use of ΔV neglects any transmitted DC component. For NRZ (non-return to zero, see section 6.4), unipolar, rectangular pulse signalling, Figure 6.4, the normalised peak signal power is $S_{peak} = \Delta V^2$ and the average signal power is $S = \Delta V^2/2$. The normalised Gaussian noise power is $N = \sigma^2$. We therefore have:

$$\frac{\Delta V}{\sigma} = \left(\frac{S}{N}\right)_{peak}^{1/2} = \sqrt{2}\left(\frac{S}{N}\right)^{1/2} \tag{6.9}$$

where $(S/N)_{peak}$ indicates peak signal power divided by average (or expected) noise power. (The peak noise power would in principle be infinite for Gaussian noise.) Substituting for $\Delta V/\sigma$ in equation (6.8) we therefore have:

[2] The Q-function, which represents the area under the tail of a (zero mean, unit variance) Gaussian pdf, is defined by $Q(z) = \int_z^\infty (1/\sqrt{2\pi}) e^{-(x^2/2)}\, dx$. This function is often used as an alternative to erfc(z) in the formulation of P_e problems and is related to it by $Q(z) = \frac{1}{2}\text{erfc}(z/\sqrt{2})$ or erfc $(z) = 2Q(z\sqrt{2})$.

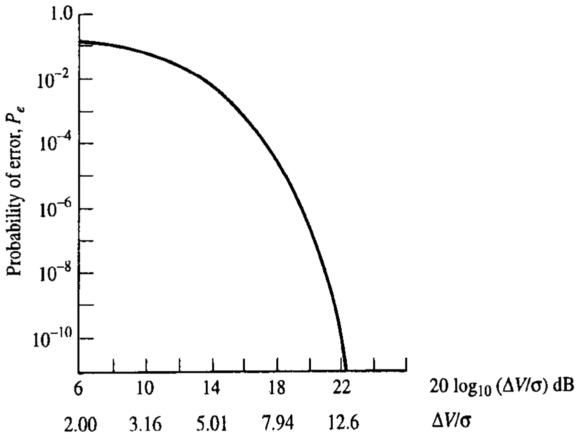


Figure 6.3 *Probability of error versus $\Delta V/\sigma$ (polar NRZ SNR = $20\log_{10}(\Delta V/\sigma) - 6dB$, while unipolar SNR = $20\log_{10}(\Delta V/\sigma) - 3$ dB).*

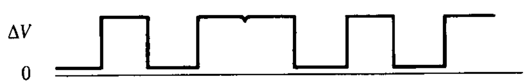


Figure 6.4 *Unipolar rectangular pulse signal.*

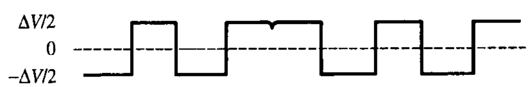


Figure 6.5 *Polar rectangular pulse signal.*

$$P_e = \frac{1}{2}\left[1 - \text{erf}\,\frac{1}{2\sqrt{2}}\left(\frac{S}{N}\right)_{peak}^{1/2}\right] \tag{6.10(a)}$$

or

$$P_e = \frac{1}{2}\left[1 - \text{erf}\,\frac{1}{2}\left(\frac{S}{N}\right)^{1/2}\right] \tag{6.10(b)}$$

For NRZ, polar, rectangular pulse signalling with the same voltage spacing as in the unipolar case, Figure 6.5, the peak and average signal powers are identical, i.e. $S_{peak} = S = (\Delta V/2)^2$ and we can therefore write:

$$\frac{\Delta V}{\sigma} = 2\left(\frac{S}{N}\right)_{peak}^{1/2} = 2\left(\frac{S}{N}\right)^{1/2} \tag{6.11}$$

Substituting into equation (6.8) we now have:

$$P_e = \frac{1}{2}\left[1 - \text{erf}\,\frac{1}{\sqrt{2}}\left(\frac{S}{N}\right)^{½}\right] \tag{6.12}$$

where a distinction between peak and average SNR no longer exists. Equations (6.10) and (6.12) are specific to a particular signalling format. In contrast equation (6.8) is general and, therefore, more fundamental. For statistically independent, equiprobable binary symbols, such as are being discussed here, each symbol carries one bit of information (see Chapter 9). The probability of symbol error, P_e, in this special case is therefore identical to the probability of bit error, P_b.

Whilst probabilities of symbol and bit error are the quantities usually *calculated*, it is the symbol error rate (SER) or bit error rate (BER) which is usually *measured*. These are simply the number of symbol or bit errors occurring per unit time (usually one second) measured over a convenient (and sometimes specified) period. The symbol error rate is clearly related to the probability of symbol error by:

$$\text{SER} = P_e R_s \tag{6.13}$$

where R_s is the symbol rate in symbol/s or baud. Bit error rate is related to the probability of bit error by:

$$\text{BER} = P_b R_s H = P_b R_b \tag{6.14}$$

where H, the entropy of the source (see Chapter 9), is the average number of information bits carried per symbol and R_b is the information bit rate.

EXAMPLE 6.1

Find the BER of a 100 kbaud, equiprobable, binary, polar, rectangular pulse signalling system assuming ideal centre point decisions, if the measured SNR at the detector input is 12.0 dB.

$$\frac{S}{N} = 10^{\frac{12}{10}} = 15.85$$

Using equation (6.12):

$$P_e = P_b = \frac{1}{2}\left[1 - \text{erf}\,\frac{1}{\sqrt{2}}(15.85)^{½}\right]$$

$$= \frac{1}{2}\,[1 - \text{erf}\,(2.815)] = 3.45 \times 10^{-5}$$

Using equation (6.14):

$$\text{BER} = P_b R_s H$$

$$= 3.45 \times 10^{-5} \times 100 \times 10^3 \times 1 = 3.45 \text{ bit errors/s}$$

6.2.2 Multilevel baseband signalling

Figure 6.6 shows a schematic diagram of a multilevel (or multisymbol) signal. If the number of, equally spaced, allowed levels is M then the symbol plus noise pdfs look like those in Figure 6.7 (drawn for $M = 4$). The probability of symbol error for the $M-2$ inner symbols (i.e. any but the symbols represented by the lowest or highest voltage level) is now twice that in the binary case. This is because the symbol can be in error if the signal plus noise voltage is too high *or* too low, i.e.:

$$P_{eM}\big|_{inner\ symbols} = 2P_e \tag{6.15}$$

The symbol error for the two outer levels (i.e. the symbols represented by the lowest and highest voltage levels) is identical to that for the binary case, i.e.:

$$P_{eM}\big|_{outer\ symbols} = P_e \tag{6.16}$$

Once again assuming equiprobable symbols, the average probability of symbol error is:

$$\begin{aligned} P_{eM} &= \frac{M-2}{M}\,2P_e + \frac{2}{M}\,P_e \\ &= \frac{2(M-1)}{M}\,P_e \end{aligned} \tag{6.17}$$

Substituting for P_e from equation (6.8) we have:

$$P_{eM} = \frac{M-1}{M}\left[1 - \mathrm{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right] \tag{6.18}$$

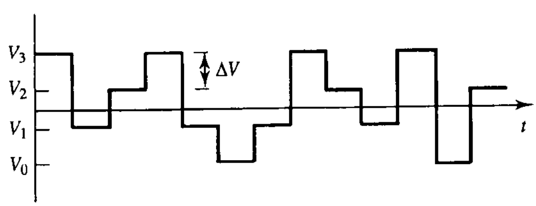


Figure 6.6 *Illustration of waveform for four level baseband signalling.*

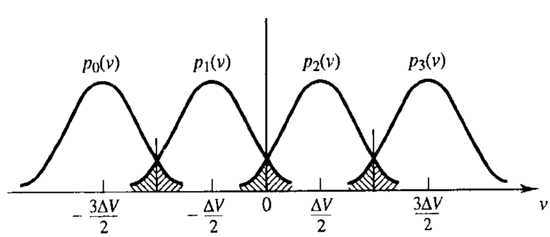


Figure 6.7 *Conditional pdfs for the four level baseband signalling system in Figure 6.6 assuming Gaussian noise ($p_i(v) = p(v|V_i)$).*

where ΔV is the difference between the equally spaced, adjacent, voltage levels, Figure 6.6. (Multilevel signalling is extended from baseband to bandpass systems in Chapter 11 where the different symbols are represented by differences in signal frequency, amplitude or phase.)

EXAMPLE 6.2

A four level, equiprobable, baseband signalling system uses NRZ rectangular pulses. The attenuation between transmitter and receiver is 15 dB and the noise power at the 50 Ω input of an ideal centre point decision detector is 10 μW. Find the average signal power which must be transmitted to maintain a symbol error probability of 10^{-4}.

The standard deviation of the noise voltage is equal to the RMS noise voltage (since the noise has zero mean).

$$\sigma = \sqrt{P\,R}$$

$$= \sqrt{1 \times 10^{-5} \times 50} = 2.236 \times 10^{-2} \quad \text{(V)}$$

Rearranging equation (6.18):

$$\Delta V = 2\sigma \sqrt{2}\, \text{erf}^{-1}\left[1 - \frac{M\,P_{eM}}{M-1}\right]$$

$$= 2 \times 2.236 \times 10^{-2} \times \sqrt{2}\ \text{erf}^{-1}\left[1 - \frac{4 \times 10^{-4}}{4-1}\right]$$

$$= 6.324 \times 10^{-2}\ \text{erf}^{-1}(0.999867) = 0.171 \quad \text{(V)}$$

Thus symbol levels are ±85.5 mV, ±256 mV in Figures 6.6 and 6.7.

Two of the symbols are represented by received power levels of $0.0855^2/50 = 1.46 \times 10^{-4}$ W and two by received power levels of $0.256^2/50 = 1.31 \times 10^{-3}$ W. Since the symbols are equiprobable the average received power, S_R, must be:

$$S_R = \tfrac{1}{2}(0.146 + 1.31)10^{-3}\ \text{W} = 0.728\ \text{mW}$$

The transmitted power, P_T, is therefore:

$$P_T = S_R \times 10^{\frac{15}{10}}$$

$$= 0.728 \times 31.62 = 23\ \text{mW}$$

6.3 Error accumulation over multiple hops

All signal transmission media (e.g. cables, waveguides, optical fibres) attenuate signals to a greater or lesser extent. This is even the case for the space through which radio waves travel (although for free space, the use of the word attenuate, to describe this effect, might be disputed – see Chapters 12 and 14). For long communication paths attenuation might be so severe that the sensitivity of normal receiving equipment would be inadequate to detect the signal. In such cases the signal is boosted in amplitude at regular intervals along the transmitter–receiver path. The equipment which boosts the signal is called a *repeater* and the path between adjacent repeaters is called a *hop*. (A repeater along with its associated, preceding, hop is called a *section*.) Thus long distance communication is usually achieved using multiple hops (see also later, section 19.4).

Repeaters used on multihop links can be divided into essentially two types. These are *amplifying* repeaters and *regenerative* repeaters. For analogue communications linear amplifiers are required. For digital communications either type of repeater could be used but normally regenerative repeaters are employed.

Figure 6.8 shows a schematic diagram of an m-hop link. (m hops imply a transmitter, a receiver and $m-1$ repeaters.) If a binary signal with voltage levels $\pm\Delta V/2$ is transmitted then the voltage received at the input of the first amplifying repeater is $\pm\alpha\Delta V/2 + n_1(t)$ where α is the linear voltage attenuation factor (or voltage gain factor < 1) and $n_1(t)$ is the random noise (with standard deviation, or RMS value, σ) added during the first hop. In a well designed system the voltage gain, G_V, of the repeater will be just adequate to compensate for the attenuation over the first hop, i.e. $G_V = 1/\alpha$. At the output of the first repeater the signal is restored to its original level (i.e. $\pm\Delta V/2$) but the noise signal is also amplified to a level $G_V n_1(t)$ and will now have an RMS value of $G_V\sigma$. (Chapter 12 contains a discussion on the origin of such noise.)

Assuming that each hop incurs the same attenuation the signal voltage at the input to the second repeater will again have fallen to $\pm\alpha\Delta V/2$ and the noise voltage from the first hop will have fallen to $n_1(t)$. A similar noise voltage, $n_2(t)$, arising from the second hop will also, however, be added and providing it is statistically independent from $n_1(t)$ will add to it on a power basis. The total noise power (in 1 Ω) will therefore be the sum of the noise variances, i.e. $2\sigma^2$ assuming equal noise is added on each hop. It can therefore be seen that with amplifying repeaters the noise power after m hops will be m times the noise power after one hop whilst the signal voltage at the receiver will be essentially the same as at the input to the first repeater. The probability of error for an m-hop link is therefore given by equation (6.8) with the RMS noise voltage, σ, replaced by $\sigma\sqrt{m}$, i.e.:

$$P_e\big|_{m\ hops} = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2m}}\right)\right] \tag{6.19}$$

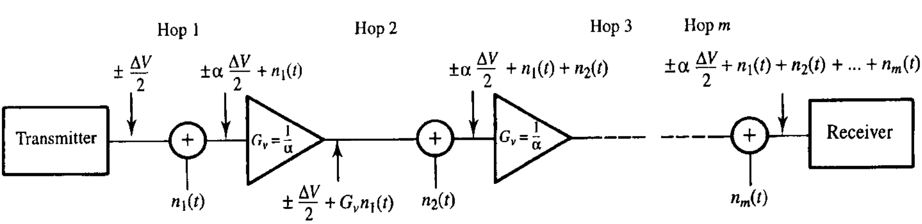


Figure 6.8 *Multihop link utilising linear amplifiers as signal boosters.*

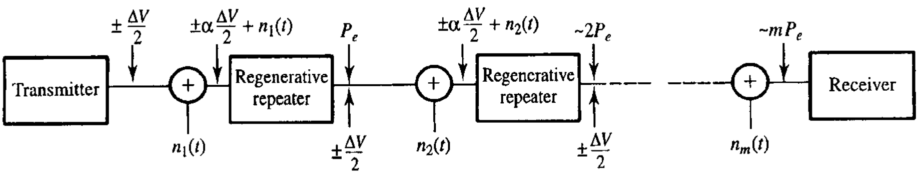


Figure 6.9 *Multihop link utilising regenerative repeaters as signal boosters.*

If the linear amplifiers are replaced with regenerative repeaters, Figure 6.9, the situation changes dramatically. The repeater now uses a decision process to establish whether a digital 0 or 1 is present at its input and a new, and noiseless, pulse is generated for transmission to the next repeater. Noise does not therefore accumulate from repeater to repeater as it does in the case of linear amplifiers. There is, however, an equivalent process in that symbols will be detected in error at each repeater with a probability P_e given by equation (6.8). Providing this probability is small (specifically $mP_e \ll 1$) the probability of any given symbol being detected in error (and therefore inverted) more than once over the m-hops of the link can be neglected. In this case the probability of error (rather than the noise power) accumulates linearly over the hops and after m hops we have:

$$P_e\big|_{m\ hops} = mP_e = \frac{m}{2}\left[1 - \text{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right] \tag{6.20}$$

where P_e is the one-hop probability given in equation (6.8).

Figure 6.10(a) illustrates the increase in $P_e\big|_{m\ hops}$ as the number of hops increases for both amplifying and regenerative repeaters, clearly showing the benefit of digital regeneration. Figure 6.10(b) shows the typical saving in transmitter power (per repeater) realised using digital regeneration. (The power saving shown is the square of the factor by which ΔV in equation (6.19) must be larger than ΔV in equation (6.20) for a $P_e = 10^{-5}$ after m hops.)

EXAMPLE 6.3

If 15 link sections, each identical to that described in Example 6.1, are cascaded to form a 15-hop link find the probability of bit error when the repeaters are implemented as (i) linear amplifiers and (ii) digital regenerators.

(i) Using equation (6.11):

$$\frac{\Delta V}{\sigma} = 2\left(\frac{S}{N}\right)^{½} = 2(15.85)^{½} = 7.962$$

Now using equation (6.19):

$$P_e = \frac{1}{2}\left[1 - \text{erf}\left(\frac{7.962}{2\sqrt{2}\sqrt{15}}\right)\right]$$

$$= \tfrac{1}{2}\ [1 - \text{erf}\ (0.727)] = 0.152$$

(ii) Since $mP_e = 15 \times 3.45 \times 10^{-5} << 1.0$ we can use equation (6.20), i.e.:

$$P_e\big|_{15\ hops} \approx 15 \times P_e\big|_{1\ hop} = 5.175 \times 10^{-4}$$

The advantage of regenerative repeaters in this example is clear.

6.4 Line coding

The discussion of baseband transmission up to now has centred on the BER performance of unipolar, and polar, rectangular pulse representations of the binary symbols zero and one. Binary data can, however, be transmitted using many other pulse types. The choice of a particular pair of pulses to represent the symbols 1 and 0 is called line coding and

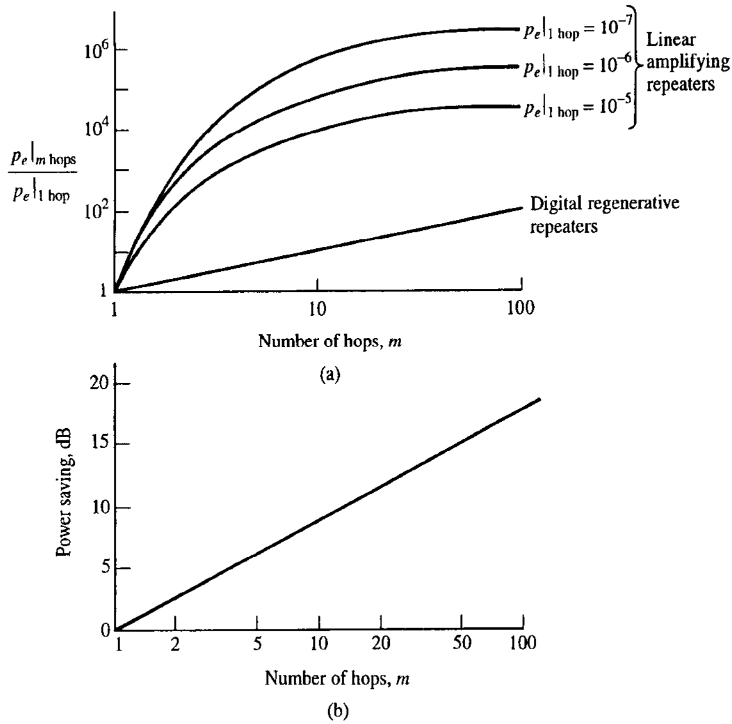


Figure 6.10 *(a) P_e degradation due to multiple hops for different repeater types (source: Stremler, 1990, reproduced with the permission of Addison Wesley) and (b) saving in transmission power using digital regenerative, instead of linear, repeaters for P_e = 10^{-5} (source: Carlson, 1986, reproduced with the permission of McGraw-Hill).*

selection is usually made on the grounds of one or more of the following considerations:

1. Presence or absence of a DC level.
2. Power spectral density – particularly its value at 0 Hz.
3. Spectral occupancy (i.e. bandwidth).
4. BER performance (i.e. relative immunity from noise).
5. Transparency (i.e. the property that any arbitrary symbol, or bit, pattern can be transmitted and received).
6. Ease of clock signal recovery for symbol synchronisation.
7. Presence or absence of inherent error detection properties.

Line coding is usually thought of as the selection, or design, of pulse pairs which retain sharp transitions between voltage levels. Figure 6.11, in which T_o represents the symbol period, shows a variety of pulse types and spectra corresponding to several commonly used line codes. Figure 6.12 shows the transmitted waveforms of these (and a few other) line codes for an example sequence of binary data and Table 6.1 compares some of their important properties.

After line coding the pulses may be filtered or otherwise shaped to further improve their properties, for example their spectral efficiency and/or immunity to intersymbol interference (see Chapter 8). The distinction between line coding and pulse shaping is not always easy to make, and in some cases, it might be argued, artificial. Here, however, we make the distinction if for no other reason than to subdivide our discussion of baseband transmission into manageable parts. The line codes included in Figures 6.11

Table 6.1 *Comparison of line (baseband) code performance.*

	Timing extraction	*Error detection*	*Relative transmitter power (single-point decisions)*		*First null bandwidth*	*AC coupled*	*Trans-parent*
			Average	*Peak*			
Unipolar (NRZ)	Difficult	No	2	4	f_o	No	No
Unipolar (RZ)	Simple	No	1	4	$2f_o$	No	No
Polar (NRZ)	Difficult	No	1	1	f_o	No	No
Polar (RZ)	Rectify	No	½	1	$2f_o$	No	No
Dipolar - OOK	Simple	No	2	4	$2f_o$	Yes	No
Dipolar - split ϕ	Difficult	No	1	1	$2f_o$	Yes	Yes
Bipolar (RZ)	Rectify	Yes	1	4	f_o	Yes	No
Bipolar (NRZ)	Difficult	Yes	2	4	$f_o/2$	Yes	No
HDB3	Rectify	Yes	1	4	f_o	Yes	Yes
CMI	Simple	Yes	See note	See note	$2f_o$	Yes	Yes

Note for CMI transmission at least two samples per symbol are required (e.g. taken 0.25 and 0.75 of the way through the symbol pulse). The difference between these samples results in a detected voltage which is twice the transmitted level for a digital zero, and a voltage of zero for a digital one. The RMS noise, σ, is increased by a factor of $\sqrt{2}$ due to the addition of the independent noise samples. On the (same) equal BER basis as the required transmitter powers given in Table 6.1 this results in a required CMI relative transmitter power (both average and peak) of 2. To compare this fairly with the other line codes, the relative powers required in Table 6.1 must all be reduced by a factor of 0.5 to reflect the factor of 2 improvement in SNR due to double sampling.

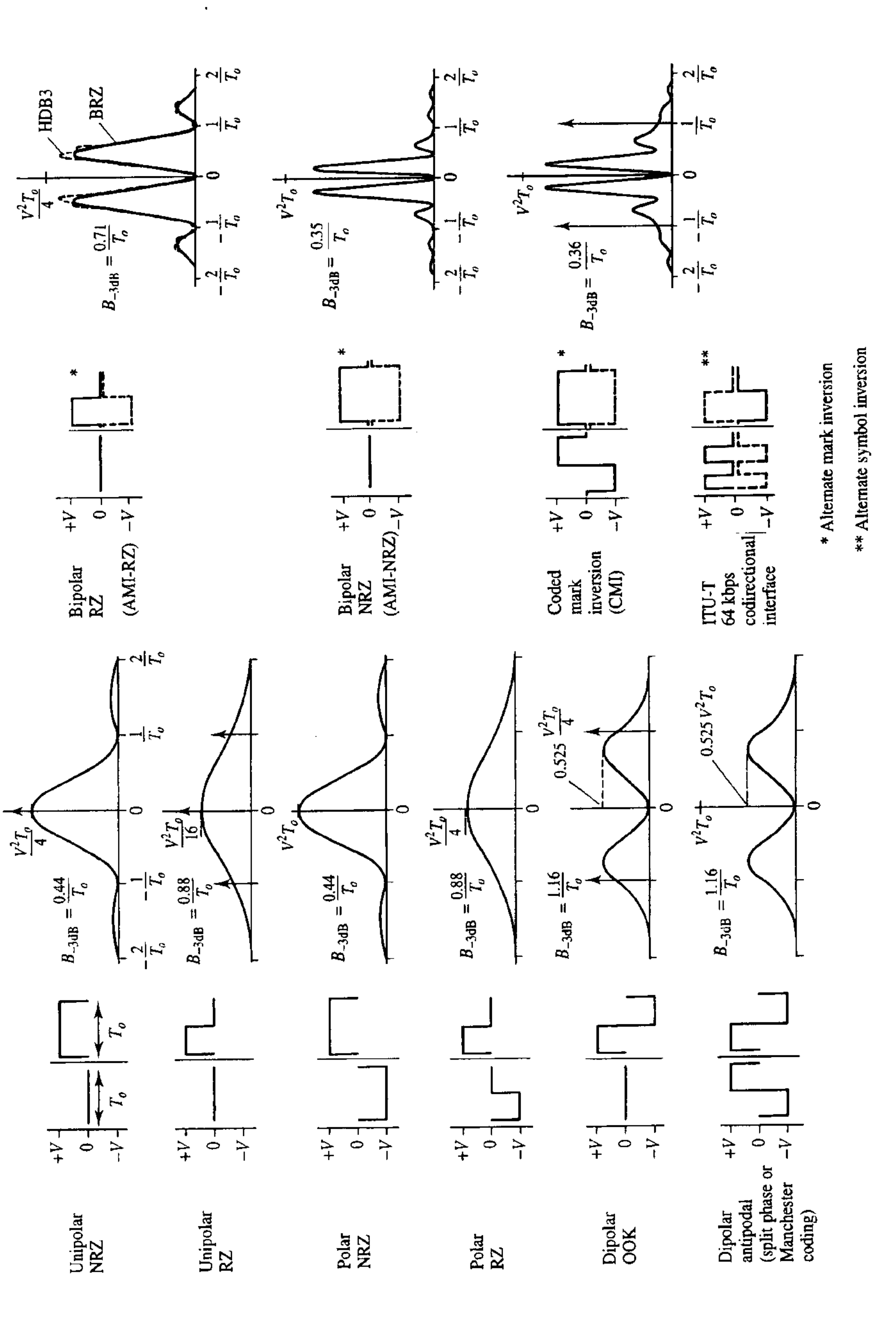


Figure 6.11 *Selection of commonly used line code symbols (0, 1) and associated spectra.*

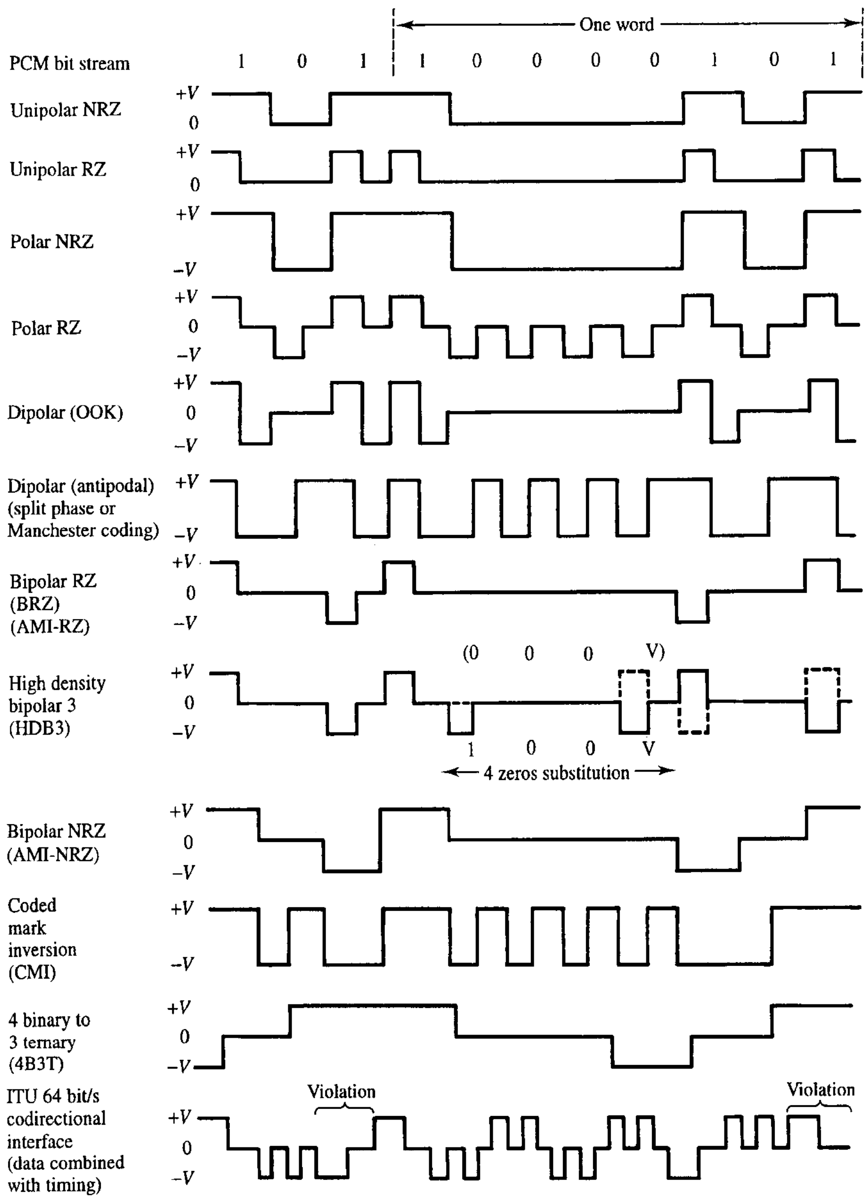


Figure 6.12 *Various line code waveforms for PCM bit sequence.*

and 6.12, and Table 6.1, do not form a comprehensive list, there being many other baseband signalling formats. One notable line code not included here, for example, is the Miller code which has a particularly narrow spectrum centred on $0.4/T_o$ Hz [Stremler, Sklar].

For the simple binary signalling formats (with statistically independent symbols), shown on the left hand side of Figure 6.11, the power spectral densities can be found using [Stremler]:

$$G(f) = p(1-p)\frac{1}{T_o}|F_1(f) - F_2(f)|^2 + \frac{1}{T_o^2}\sum_{n=-\infty}^{\infty} |pF_1(nf_o) + (1-p)F_2(nf_o)|^2\, \delta(f - nf_o) \quad (6.21)$$

where p is the probability of symbol 1, $f_o = 1/T_o$ represents the symbol rate and $F(f)$ is the symbol voltage spectrum.

The codes which appear in Figures 6.11, 6.12 and Table 6.1 are now described.

6.4.1 Unipolar signalling

Unipolar signalling (also called on-off keying, OOK) refers to a line code in which one binary symbol (denoting a digital zero, for example) is represented by the absence of a pulse (i.e. a space) and the other binary symbol (denoting a digital one) is represented by the presence of a pulse (i.e. a mark). There are two common variations on unipolar signalling, namely non-return to zero (NRZ) and return to zero (RZ). In the former case the duration (τ) of the mark pulse is equal to the duration (T_o) of the symbol slot. In the latter case τ is less than T_o.

Typically RZ pulses fill only the first half of the time slot, returning to zero for the second half. (The mark duty cycle, τ/T_o, would be 50% in this case although other duty cycles can be, and are, used.) The power spectral densities of both NRZ and RZ signals have a $[(\sin x)/x]^2$ shape where $x = \pi\tau f$. RZ signals (assuming a 50% mark duty cycle) have the disadvantage of occupying twice the bandwidth of NRZ signals (see Figure 6.11). They have the advantage, however, of possessing a spectral line at the symbol rate, $f_o = 1/T_o$ Hz (and its odd integer multiples), which can be recovered for use as a symbol timing clock signal, Table 6.1. Non-linear processing must be used to recover a clock waveform from an NRZ signal (see section 6.7).

Both NRZ and RZ unipolar signals have a non-zero average (i.e. DC) level represented in their spectra by a line at 0 Hz, Figure 6.11. Transmission of these signals over links with either transformer or capacitor coupled (AC) repeaters results in the removal of this line and the consequent conversion of the signals to a polar format. Furthermore, since the continuous part of both the RZ and NRZ signal spectrum is non-zero at 0 Hz then AC coupling results in distortion of the transmitted pulse shapes. If the AC coupled lines behave as highpass RC filters (which is typically the case) then the distortion takes the form of an exponential decay of the signal amplitude after each transition. This effect, referred to as signal 'droop', is illustrated in Figure 6.13 for an NRZ signal. Although the long term DC component is zero, after AC coupling 'short term' DC levels accumulate with long strings of ones or zeros. The accumulated DC level is most apparent for the first few symbols after a string represented by a constant voltage. Neither variety of unipolar signal is therefore suitable for transmission over AC coupled lines.

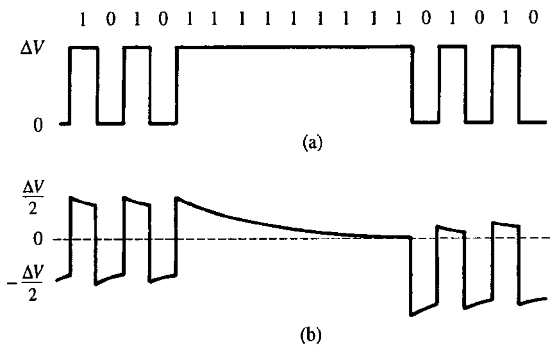


Figure 6.13 *Distortion due to AC coupling of unipolar NRZ signal: (a) input; (b) output.*

Since unipolar voltage levels of 0 or V volts are equivalent (in terms of BER) to polar levels of $\pm V/2$ volts (section 6.2.1) then unipolar signalling requires twice the average, and four times the peak, transmitter power when compared with polar signalling, Table 6.1.

6.4.2 Polar signalling

In polar signalling systems a binary one is represented by a pulse $g_1(t)$ and a binary zero by the opposite (or *antipodal*) pulse $g_0(t) = -g_1(t)$, Figure 6.11. Figure 6.12 compares polar and unipolar signals for a typical data stream. The NRZ and RZ forms of polar signals have identically shaped spectra to the NRZ and RZ forms of unipolar signals except that, due to the opposite polarity of the one and zero symbols, neither contain any spectral lines. Polar signals have the same bandwidth requirements as their equivalent unipolar signals and suffer the same distortion effects (in particular signal droop) if transmitted over AC coupled lines, Table 6.1.

As pointed out in section 6.4.1 polar signalling has a significant power (or alternatively BER) advantage over unipolar signalling. Fundamentally this is because the pulses in a unipolar scheme are only *orthogonal* whilst the pulses in a polar scheme are *antipodal.* Another way of explaining the difference in performance is to observe that the average or DC level transmitted with unipolar signals contains no information and is therefore wasted power.

Polar binary signalling also has the advantage that, providing the symbols are equiprobable, the decision threshold is 0 V. This means that no automatic gain control (AGC) is required in the receiver.

6.4.3 Dipolar signalling

Dipolar signalling is designed to produce a spectral null at 0 Hz. This makes it especially well suited to AC coupled transmission lines. The symbol interval, T_o, is split into positive and negative pulses each of width $T_o/2$ s, Figure 6.11. This makes the total area under either pulse type equal to zero which results in the desirable DC null in the signal's spectrum. Both OOK and antipodal forms of dipolar signalling are possible, the latter being called split phase or Manchester coding (Figure 6.11). A spectral line at the clock frequency ($1/T_o$ Hz) is present in the OOK form but absent in the antipodal form.

Manchester coding is widely used for the distribution of clock signals within VLSI circuits, for magnetic recording and for Ethernet LANs (see Chapter 18).

6.4.4 Bipolar alternate mark inversion signalling

Bipolar signalling (also called alternate mark inversion) uses three voltage levels ($+V$, 0, $-V$) to represent two binary symbols (0 and 1) and is therefore a pseudo-ternary line code. Zeros, as in unipolar signalling, are represented by the absence of a pulse (i.e. 0 V) and ones (or *marks*) are represented alternately by voltage levels of $+V$ and $-V$. Both RZ and NRZ forms of bipolar signalling are possible, Figure 6.11, although the RZ form is more common. Alternating the mark voltage level ensures that the bipolar spectrum has a null at DC and that signal droop on AC coupled lines is avoided. The alternating mark voltage also gives bipolar signalling a single error detection capability and reduces its bandwidth over that required for the equivalent unipolar or polar format (see Figure 6.11).

6.4.5 Pulse synchronisation and HDBn coding

Pulse synchronisation is usually required at a repeater or receiver to ensure that the samples, on the basis of which symbol decisions are made, are taken at the correct instants in time. In principle those line codes (such as unipolar RZ and dipolar OOK) which possess a spectral line at $1/T_o$ Hz have an inherent pulse synchronisation capability since all that is required to regenerate a clock signal is for this spectral line to be extracted using a filter or phase locked loop. Other line codes which do not possess a convenient spectral line can often be processed, for example by rectification, in order to generate one (see later Figures 6.26 to 6.29). This is the case for the bipolar RZ (BRZ) line code which is often used in practical PCM systems.

Although rectification of a BRZ signal results in a unipolar RZ signal and therefore a spectral line at $1/T_o$ Hz, in practice there is a problem if long strings of zeros are transmitted. In this case pulse synchronisation might be lost due, for instance, to loss of lock of the pulse timing phase locked loop, Figure 6.30. To prevent this, many BRZ systems use high density bipolar substitution (HDBn). Here, when the number of continuous zeros exceeds n then they are replaced by a special code. $n = 3$ (HDB3) is the code recommended (G.703) by ITU-T for PCM systems at multiplexed bit rates of 2, 8 and 34 Mbit/s (see section 19.2). In HDB3 a string of four zeros is replaced by either 000V or 100V. Here V is a binary 1 with sign chosen to *violate* the alternating mark rule so that it can be detected as the special sequence representing the all zero code, Figure 6.12. Furthermore, consecutive violation (V) pulses alternate in polarity to avoid introducing a DC component. (This is achieved by having the two possibilities 000V or 100V. The selection depends on the number of digital ones since the last code insertion.)

The HDB spectrum has minor variations compared with the BRZ signal from which it is derived (Figure 6.11). HDB3 is sometimes referred to as B4ZS denoting bipolar signalling with four-zeros substitution.

6.4.6 Coded mark inversion (CMI)

CMI is a polar NRZ code which uses both amplitude levels (each for half the symbol period) to represent a digital 0 and either amplitude level (for the full symbol period) to represent a digital 1. The level used alternates for successive digital ones. CMI is therefore a combination of dipolar signalling (used for digital zeros) and NRZ AMI (used for digital ones). CMI is the code recommended (G.703) by ITU-T for 140 Mbit/s multiplexed PCM (see Chapter 19). The ITU codirectional interface at 64 kbit/s uses a refinement of CMI, Figure 6.11, in which the polarity of consecutive symbols (irrespective of whether they are 1s or 0s) is alternated. Violations of the alternation rule are then used every eighth symbol to denote the last bit of each (8-bit) PCM code word, Figure 6.12.

Table 6.2 *4B3T coding example showing uncoded binary and coded ternary signals.*

Binary input signal	*Ternary output signal Running digital sum at end of preceding word equal to*	
	−2, −1 or 0	1, 2 or 3
0000	+ 0 −	+ 0 −
0001	− + 0	− + 0
0010	0 − +	0 − +
0011	+ − 0	+ − 0
0100	0 + −	0 + −
0101	− 0 +	− 0 +
0110	0 0 +	0 0 −
0111	0 + 0	0 − 0
1000	+ 0 0	− 0 0
1001	+ + −	− − +
1010	+ − +	− + −
1011	− + +	+ − −
1100	0 + +	0 − −
1101	+ 0 +	− 0 −
1110	+ + 0	− − 0
1111	+ + +	− − −

6.4.7 *n*B*m*T coding

*n*B*m*T is a line code in which *n* binary symbols are mapped into *m* ternary symbols. A coding table for $n = 4$ and $m = 3$ (i.e. 4B3T) is shown as Table 6.2. This code lengthens the transmitted symbols to reduce the signal bandwidth. In Table 6.2 the top six outputs are balanced and hence are fixed. The lower 10 have a polarity imbalance and need occasionally to be inverted to avoid the running digital sum causing DC wander. In the table the left hand column of coded signals or symbols (which sum to a zero or positive value) is used if the preceding running sum is negative. The right hand column, in which the symbols sum to zero or a negative value, is used if the preceding running sum is positive. With three ternary symbols we have $3^3 = 27$ possible states and by not transmitting (0, 0, 0) we have 26, comprising the 6 + 10 + 10 unique states in Table 6.2. As $2^4 = 16$ this conveniently matches to the 4-bit binary code requirements.

The 4B3T spectrum is further modified from that of HDB3 skewing the energy towards low frequencies and 0 Hz [Flood and Cochrane]. A further development of this concept is 2B1Q where two binary bits are converted into one four-level (quaternary) symbol. This is an example of *M*-ary (compare with *bin*ary, *tern*ary, etc.) signalling. *M*-ary coding is examined further in sections 11.4 and 11.5 in the context of carrier based coded signals.

6.5 Multiplex telephony

PCM is used in conjunction with TDM to realise multichannel digital telephony. The internationally agreed *European* ITU standard provides for the combining of 30 speech channels, together with two subsidiary channels for signal and system monitoring. Each speech channel signal is sampled at 8 kHz and non-linearly quantised (or companded) into 8-bit words (see Chapter 5). The binary symbol rate per speech channel is therefore 64 kbit/s, and for the composite 30 + 2 channel signal multiplex is 32×64 kbit/s = 2.048 Mbit/s. For convenience this is often referred to as a 2 Mbit/s signal. A key advantage of the 2 Mbit/s TDM multiplex is that it is readily transmitted over 2 km sections of twisted pair (copper) cables which originally carried only one analogue voice signal. Now, 2 Mbit/s signals are also transmitted over optical fibre circuits. In the USA and Japan the multiplex combines fewer speech channels into a 1.5 Mbit/s signal.

The 2 Mbit/s data transmission system is now examined to highlight the problems associated with transmitting and receiving such signals over a typical twisted pair, wire cable. The principal requirement is that signal fidelity should be sufficiently high to allow the receiver electronics to synchronise to the noisy and distorted incoming data signal and make correct data decisions.

6.6 Digital signal regeneration

The key facet of digital transmission systems is that after one, physically short, section in a link there is usually sufficiently high SNR to detect reliably the received binary data, and (possibly after error correction) regenerate an almost error free data stream, Figure 6.14, for retransmission over the next stage or section. Regeneration allows an increase in overall communications path length with negligible decrease in message quality provided each regenerator operates at an acceptable point on the error rate curve, Figure 6.3. If the path loss on each section is approximately 40 dB then a 3 V transmitted polar signal is received as 30 mV and, allowing for near end crosstalk noise (see later, Figure 6.23) rather than thermal noise, the SNR is typically 18 dB. At this SNR the P_e, found using equation (6.12) and the approximation:

$$\text{erf}(x) \approx 1 - \left[\frac{e^{-x^2}}{\sqrt{\pi}x}\right], \quad \text{for } x \geq 4 \tag{6.22}$$

is 10^{-15}. (For NRZ polar signalling $(\Delta V/\sigma)$ dB = $(S/N) + 6$ dB, see equation (6.11).) For identical calculations on each section then the P_e on each section will be 10^{-15} and the total error will be the sum of the errors on each section. Thus, on the two section link shown in Figure 6.14, the total error probability is 2×10^{-15}. This was discussed previously in section 6.3 where for an m-section link $(P_e|_{m\ hops})$ was shown to be m times the error on a single hop or regenerative section. (In practice the signal attenuation, and noise introduced, is rarely precisely equal for all sections. Since the slope of the P_e curve in Figure 6.3, in the normal operating region, is very steep it is often the case that the P_e performance of a multi-hop link is dominated by the performance of its worst section.)

Provided the error rate on each section is acceptable then the cumulative or summed rate for the link is low, compared with the error rate when there is no regeneration and the single section loss is very high. The input SNR must typically be 18 dB in a section design, to ensure that there is an extremely low BER on each individual section. Regeneration thus permits long distance transmission with high message quality provided the link is properly sectioned with the appropriate loss on each section. The principal

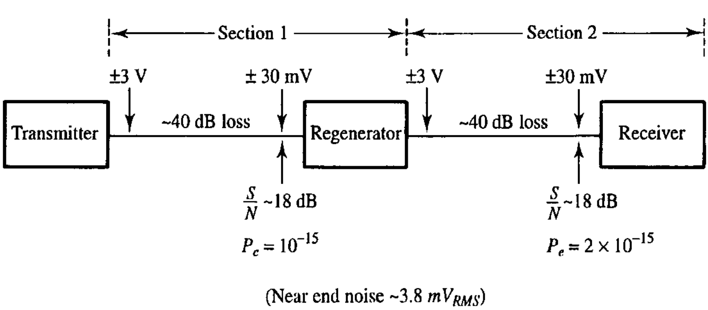


Figure 6.14 *Principle of regenerative repeater in long communications system.*

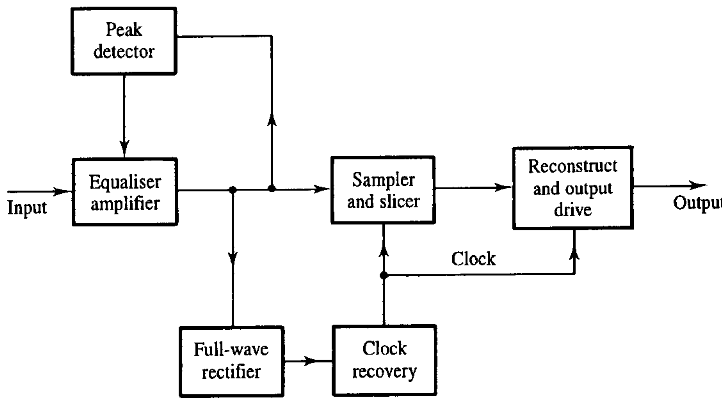


Figure 6.15 *Simplified block diagram of pulse code modulation (PCM) regenerator.*

component of such a digital line system is the digital regenerative repeater or regenerator, Figure 6.15.

6.6.1 PCM line codes

Line coding is used, primarily, to match the spectral characteristics of the digital signal to its channel and to provide guaranteed recovery of timing signals. ITU-T recommends HDB3 for (multiplexed) PCM bit rates of 2, 8 and 34 Mbit/s and CMI for 140 Mbit/s. For the unmultiplexed bit rate (64 kbit/s) ITU-T G.703 has three different line code recommendations, one for each of three PCM equipment interface standards. The code used depends on the type of equipment connected to either side of the PCM interface. Three types of equipment interface are defined, namely codirectional, centralised clock and contradirectional. The interface type depends on the origin of 64 kHz and 8 kHz timing signals and their transmission direction with respect to that of the information or data (Figure 6.16).

For a codirectional interface (Figure 6.16(a)) there is one transmission line for each transmission direction. In order to incorporate the timing signals into the data the 64 kbit/s pulse or bit period is subdivided into four 'quarter-bit' intervals. Binary ones are unipolar NRZ coded as 1100 and binary zeros are coded as 1010. The quarter-bits are then converted to a three level signal by alternating the polarity of consecutive four quarter-bit blocks. (Each quarter-bit has a nominal duration of 3.9 μs.) The alternation in polarity of the blocks is violated every eighth block. These violations represent the last PCM bit of each 8-bit PCM word (Figure 6.12).

For a centralised clock interface (Figure 6.16(b)) there is one transmission line for each transmission direction to carry PCM data and one transmission line from the central clock to the equipment on each side of the interface to carry 64 kHz and 8 kHz timing signals. The data line uses a bipolar code with a 100% duty cycle (i.e. AMI NRZ). The timing signal line code is bipolar with a duty cycle between 50% and 80% (i.e. AMI RZ)

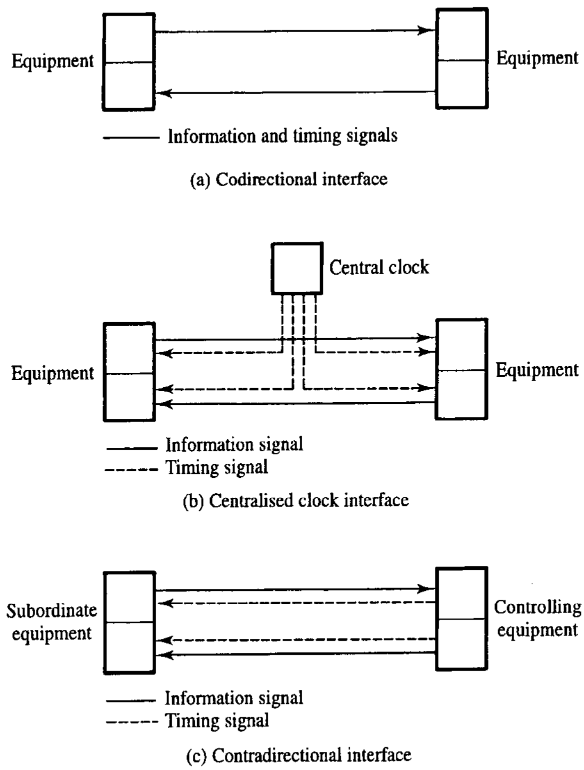


Figure 6.16 *PCM 64 kbit/s interfaces.*

with polarity violations aligned with the last (eighth) bit of each PCM code word.

For a contradirectional interface (Figure 6.16(c)) there are two transmission lines for each transmission direction, one for PCM data and one for the timing signal. The data line code (AMI NRZ) and timing signal (all ones) line code (AMI RZ) are identical to those for a centralised clock interface except that the timing signal must have a 50% duty cycle.

6.6.2 Equalisation

A significant problem in PCM cable, and many other, communication systems is that considerable amplitude and phase distortion may be introduced by the transmission medium. For a 2 Mbit/s PCM system the RZ bipolar pulse has a width of ¼ μs and hence a bandwidth of approximately 2 MHz. When this is compared with typical metallic cable characteristics, Figure 6.17, it can be seen that the received pulse will be heavily distorted and attenuated. A potentially serious consequence of this distortion is that the pulse will be stretched in time as shown in Figure 6.18.

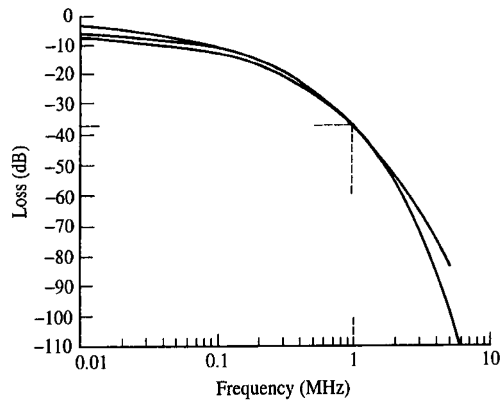


Figure 6.17 *Typical frequency responses for a 2 km length of cable.*

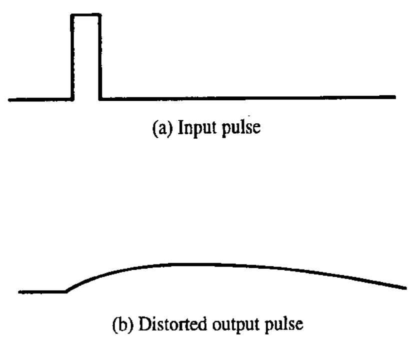


Figure 6.18 *(a) Input and (b) output 2 Mbit/s pulse for a 2 km length of cable.*

Such distortion also results when wideband signals are transmitted over the local loop telephone connection from the exchange to the subscriber. Here the cable bandwidth is only several kHz. In the high-speed digital subscriber loop (HDSL) [Baker, Young *et al.*], however, we can transmit at Mbit/s provided there is still adequate SNR at the receiver. HDSL relies on efficient data coding, such as 2B1Q, i.e. four level signalling (which is an extension of the coding ideas illustrated by Table 6.2) to reduce transmitted signal bandwidth, combined with techniques to equalise (i.e. compensate for) distortion introduced by the restricted bandwidth of the transmission network. HDSL thus achieves high rate transmission on existing networks without requiring replacement of the copper cables by alternative (optical) transmission media.

When we move from considering individual pulses to a data stream, Figure 6.19, the long time domain tails from the individual received symbols cause intersymbol interference (ISI). This is overcome by applying an equalising filter [Mulgrew and Grant] in the receiver which has the inverse frequency response, Figure 6.20, to the raw line or channel characteristic of Figure 6.17. Cascading the effect of the line with the equaliser

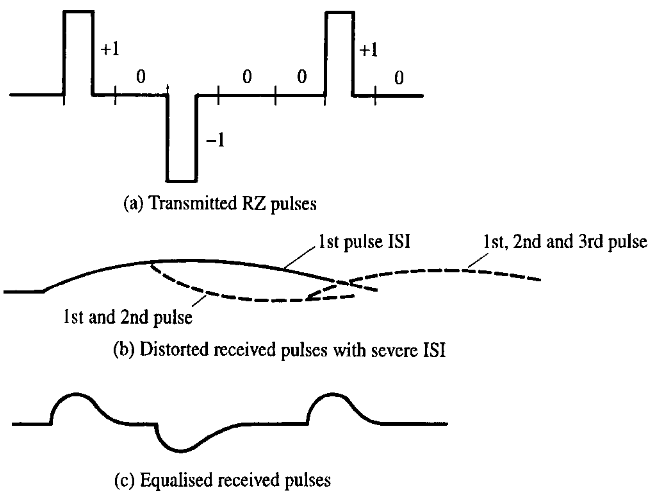


Figure 6.19 *Intersymbol interference in a pulse train.*

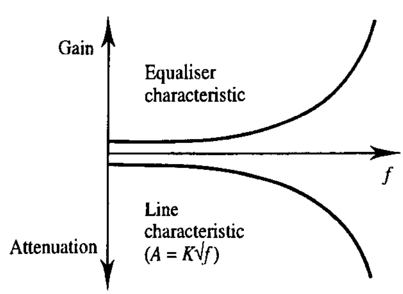


Figure 6.20 *Line equaliser frequency responses.*

provides a flat overall response which reduces the distortion (time stretching) of the pulses as shown in Figure 6.19(c). A detailed discussion of ISI and one aspect of equalisation (Nyquist filtering), which is used to minimise it, is given in Chapter 8.

6.6.3 Eye diagrams

A useful way of quickly assessing the adequacy of a digital communications system is to display its *eye diagram* on an oscilloscope. This is achieved by triggering the oscilloscope with the recovered symbol timing clock signal and displaying the symbol stream using a sweep time sufficient to show 2 or 3 symbol periods. Figure 6.21(a) shows a noiseless, but distorted, binary bit stream and Figure 6.21(b) shows the resulting two level eye pattern when the pulses are overlaid on top of one another. Distortion of the digital signal causes ISI which results in variations of the pulse amplitudes at the sampling instants. The eye opening indicates how much tolerance the system possesses

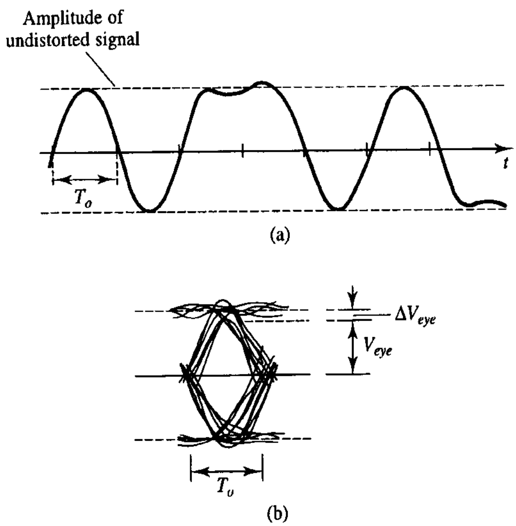


Figure 6.21 *(a) Noiseless but distorted signal; and (b) corresponding eye diagram.*

to noise before incorrect decisions will be made.

It can be seen that there is no ISI free instant in this case which could be chosen for ideal sampling. In addition to partial closing of the eye, distortion also results in a narrowing of the eye in the horizontal (time) dimension. This is because the symbol timing signal is derived from the zero crossings of the distorted bit stream resulting in symbol timing *jitter*. This effectively means that some symbols will be sampled at non-optimum instants. Figure 6.22 shows examples of eye patterns for NRZ binary and multilevel systems. The optimum sampling time clearly occurs at the position of maximum eye opening. Both distortion of the digital signal and additive noise contribute to closing of the eye. The slope of the eye pattern between the maximum eye opening and the eye corner is a measure of sensitivity to timing error.

The trade-off possible between ISI and noise can be estimated from the noise free eye diagram. For example, in Figure 6.21(b) the degradation in eye opening, $\Delta V_{eye}/V_{eye}$, is about 20%. (Notice that ΔV_{eye} is about half the thickness of the eye pattern at the point of maximum opening because distortion results in traces above as well as below the undistorted signal level.) If the signal voltage is increased by a factor $1/(1 - 0.2) = 1.25$ then the eye opening will be restored to the value it would have in the absence of ISI. An increase in SNR of $20 \log_{10} 1.25 = 1.9$ dB would therefore, at least approximately, compensate the BER degradation due to ISI.

In a practical regenerator, the major contribution to intersymbol interference generally arises from the residual amplitude of the pulse at the preceding and succeeding sampling instants and intersymbol interference from other sampling instants can usually be ignored.

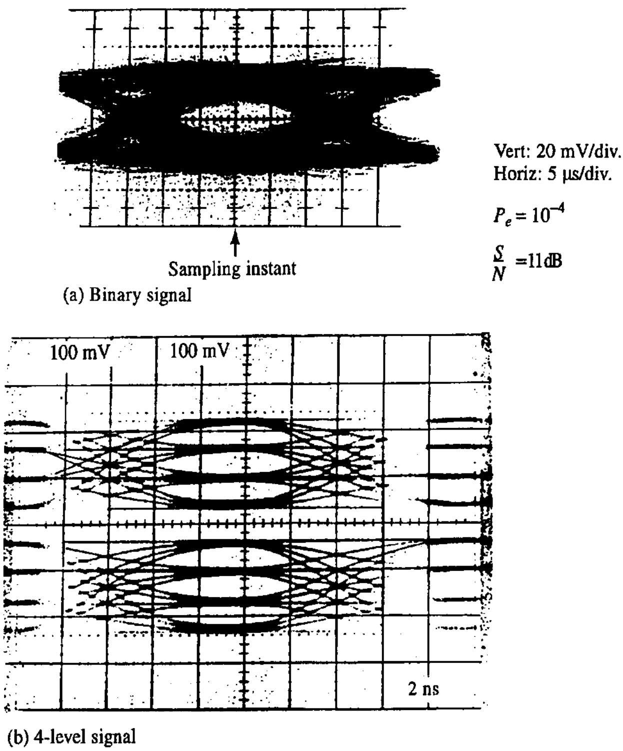


Figure 6.22 *(a) Eye diagram for NRZ digital binary signal at $P_e = 10^{-4}$ and SNR of 11 dB (source: Feher, 1983, reproduced with the permission of Prentice Hall); and (b) 4-level signal at 200 Mbit/s (source: Feher, 1981, reproduced with his permission).*

Before regenerator design is examined in more detail we need to discuss one further impairment, i.e. that caused by signal crosstalk.

6.6.4 Crosstalk

Two types of crosstalk arise when bidirectional signals are transmitted across a bundled cable comprising many individual twisted pairs. With approximately 40 dB of insertion loss across the cable a 3 V transmitted pulse is received as a 30 mV signal at the far end. Near end crosstalk (NEXT) results from the capacitive coupling of the 3 V transmitted pulse on an outgoing pair interfering with the 30 mV received pulse on an incoming pair, Figure 6.23.

Far end crosstalk (FEXT), on the other hand, occurs due to coupling of a transmitted pulse on one outgoing pair with a pulse on another outgoing pair. NEXT thus refers to crosstalk between signals travelling in opposite directions and *effectively* takes place near the cable ends whilst FEXT refers to crosstalk between signals travelling in the same

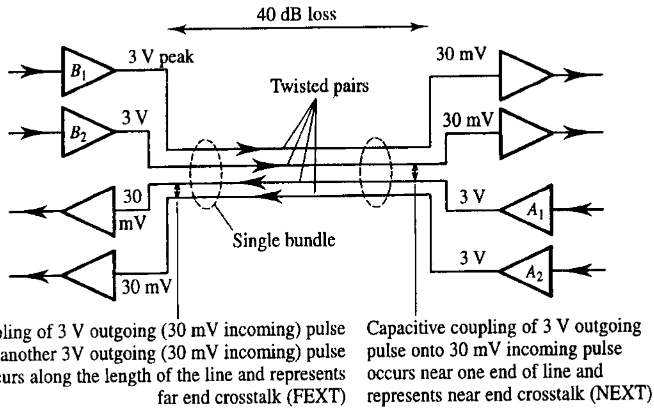
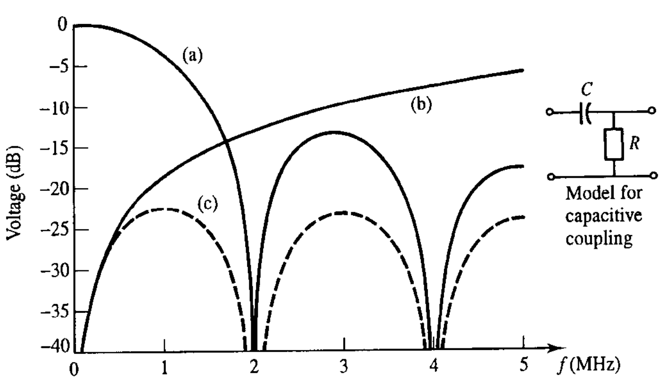


Figure 6.23 *Near and far end crosstalk (NEXT and FEXT) due to capacitive cable coupling.*

direction and takes place over the cable's entire length.

Assuming the transmitted pulse spectrum has a $|(\sin x)/x|$ shape and the coupling can be modelled as a high pass RC filter, which introduces a coupling gain of 6 dB/octave, then NEXT has a distorted spectrum as shown in Figure 6.24, in which the spectral magnitude, relative to the normal spectrum, increases with increasing frequency.

Crosstalk and ISI reduction therefore demands a composite equaliser response which differs from Figure 6.20 as shown in Figure 6.25. This has a low frequency portion



(a) Transmitted spectrum, $V_{dB} = 20 \log_{10} \left| \frac{\sin(2\pi T_o f)}{2\pi T_o f} \right|$, $T_o = 0.25$ μs

(b) Coupling loss (reducing at 6 dB/octave for high frequency), $L = 20 \log_{10} \left(\frac{2\pi f RC}{\sqrt{1 + (2\pi f RC)^2}} \right)$, $RC = 0.02$ μs

(c) Resultant NEXT spectrum, $V_{dB} + L$

Figure 6.24 *Spectrum of NEXT crosstalk signal.*

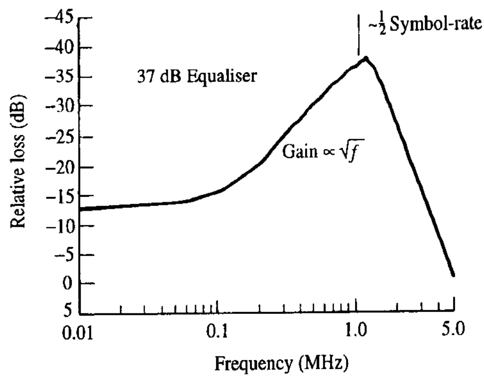


Figure 6.25 *Combined equaliser response for ISI and crosstalk.*

where loss decreases with the square root of frequency, as in Figure 6.20. The high frequency portion of the spectrum, however, is shaped to reduce crosstalk effects. This is a compromise, as it does increase ISI and sensitivity to jitter. The equaliser normally has lowest loss at half the symbol rate (i.e. 1 MHz on a 2 Mbit/s link).

6.7 Symbol timing recovery

After effective equalisation has been implemented it is necessary to derive a receiver clock, from the received signal, in order to time, accurately, the sampling and data recovery process. Symbol timing recovery (STR) is required since most digital systems are self-timed from the received signal to avoid the need for a separate timing channel. It can be achieved by first filtering and then rectifying, or squaring, a bipolar RZ line coded signal, Figure 6.26. Rectifying or squaring removes the alternating pulse format which approximately doubles the received signal's bandwidth. It removes the notch at the symbol rate, $f_o = 1/T_o$, and introduces an f_o clock signal component which can be extracted with a resonant circuit.

As the resonant circuit oscillates at its natural frequency it fills in the gaps left by the zero data bits for which no symbol is transmitted, Figure 6.27. The frequency and phase of this recovered clock must be immune from transmission distortions in, and noise on, the received signal. A problem arises in the plesiochronous multiplex in that extra (justification) bits are added or removed at the individual multiplexers to obtain the correct bit rates on the transmission links (see Chapter 19). This means that symbol timing can become irregular introducing timing jitter. Timing jitter manifests itself as FM modulation on the recovered clock signal, Figure 6.28.

The passive, tuned *LC* circuit, Figure 6.29, has a low Q factor (30 – 100) which does not give good noise suppression. Its wide bandwidth, however, means it is relatively tolerant to small changes in precise timing of the received signals (i.e. jitter). A high Q (1000 to 10000) phase locked loop gives good noise reduction but is no longer so

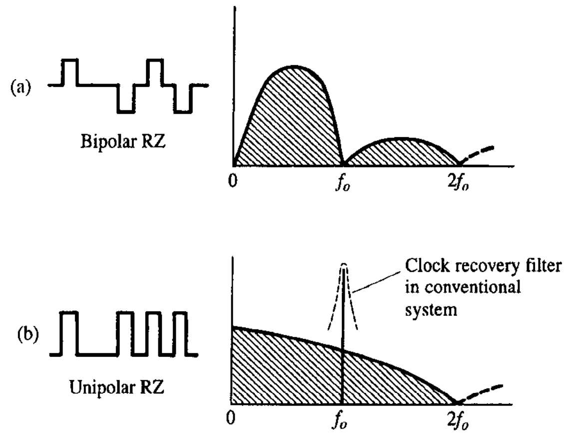


Figure 6.26 *Clock recovery by full wave rectification.*

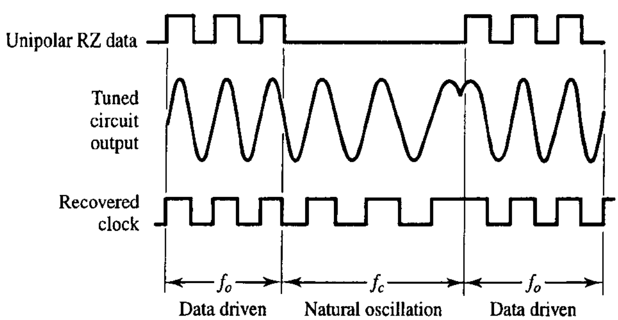


Figure 6.27 *Clock recovery oscillator operation when a string of no symbols (spaces) is received.*

tolerant to jitter effects. In Figure 6.28 we see suppression of the high frequency jitter terms. In both systems an output comparator clips the sinusoid to obtain the recovered clock waveform.

In some systems timing extraction is obtained by detecting the zero crossings of the waveform. The 'filter and square' operation is also used for STR in QPSK receivers, see later Figure 11.24. (Another QPSK STR technique is to delay the received symbol stream by half a symbol period and then form a product with the undelayed signal. This gives a periodic component at the symbol rate which can be extracted using a PLL.)

We may define peak-to-peak jitter (in seconds, or bits) as the maximum peak-to-peak displacement of a bit or symbol with respect to its position in a hypothetical unjittered reference stream. One effect of the jitter is that the regenerated clock edge varies with respect to the correct timing point in the eye diagram, thus increasing the P_e. Although noise can be removed by clipping, the remaining phase modulation still contributes to the jitter.

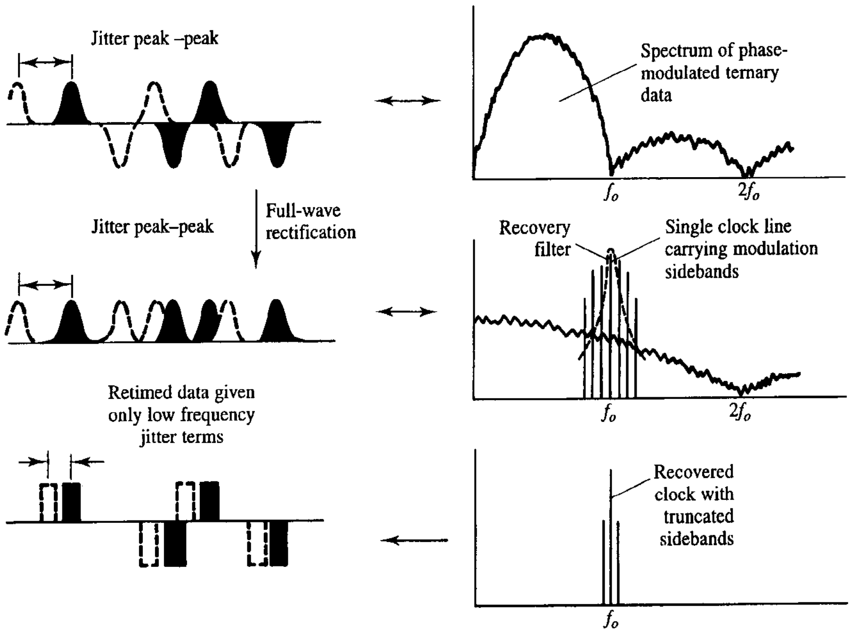


Figure 6.28 *Effect of jitter on clock recovery.*

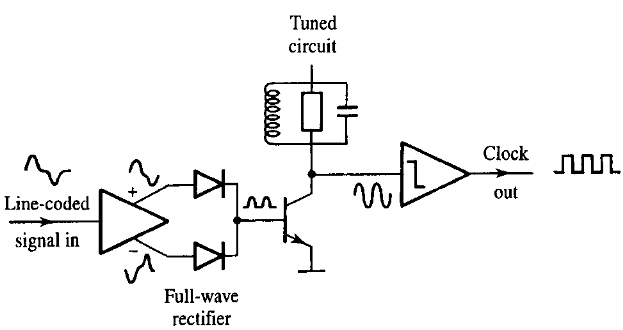


Figure 6.29 *Passive oscillator for clock recovery.*

A significant problem with jitter is that it accumulates over a multihop system. Timing jitter in the incoming data introduces eye jitter, while the clock recovery process in each repeater introduces more clock edge jitter. To minimise this problem data scramblers can be used to stop the accumulation of data dependent jitter. Limits on jitter are specified in the ITU-T G.823/4 recommendations.

6.8 Repeater design

In the receiver a matched filter (see section 8.3) for data bit detection is usually by far the most computationally demanding task. It is typically implemented using a finite impulse response filter [Mulgrew and Grant] to obtain the linear phase requirement. In comparison with this, timing extraction, phase/frequency error determination and correction usually account for only 10 to 20% of the computational load in the receiver.

A more detailed block diagram than that shown in Figure 6.15, for a complete PCM regenerative repeater, is shown in Figure 6.30. Power is fed across the entire PCM link, and each repeater AC couples the HDB3 signal, with the power extracted, from the primary winding of the coupling transformer. (The duplicated sampling and reconstruction stages in Figure 6.30 accommodate separately the positive and negative bipolar pulses.) For an m repeater link total supply voltage is m times the single repeater requirement, e.g. $m \times 5$ V. Complete repeaters are now available as single integrated circuits which only require transformer connection to the PCM system.

6.9 Summary

The simplest form of baseband digital detection uses *centre point sampling* to reduce the received symbol plus noise to a single voltage, and comparators to test this voltage against appropriate references. A formula giving the single hop probability of error for symbols represented by uniformly spaced voltages in the presence of Gaussian noise has been derived. The probability of error after *m identical* hops is *m* times greater than that after a single hop providing that *regenerative* repeaters are used between hops. The main polar and bipolar signalling techniques have been outlined and those which are preferred for various transmission systems, used in the PCM multiplex hierarchy, discussed. This

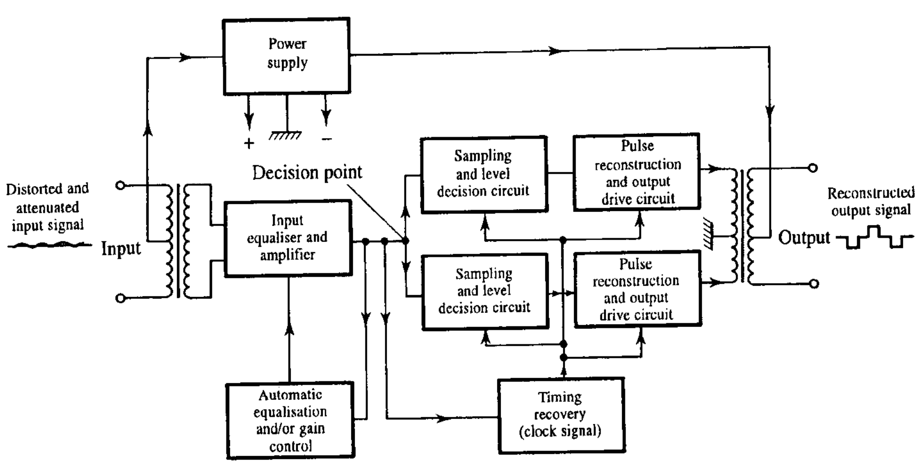


Figure 6.30 *Complete PCM regenerator.*

has involved consideration of the spectral properties of these line codes to ensure that they adequately match the transmission channel frequency response.

This chapter has also addressed some specific problems of data transmission in a metallic wire, twisted pair, cable. Distortion due to the cable frequency response and its compensation via equalisation have been examined. The effects of crosstalk interference and imperfect timing recovery have also been discussed to show their importance in practical data transmission systems. Although we have principally addressed the 2 Mbit/s metallic wire system, these signal degradation mechanisms are present in practically all digital communication systems.

6.10 Problems

6.1. A, baseband, NRZ, rectangular pulse signal is used to transmit data over a single section link. The binary voltage levels adopted for digital 0s and 1s are −2.5 V and +3.5 V respectively. The receiver uses a centre point decision process. Find the probability of symbol error in the presence of Gaussian noise with an RMS amplitude of 705 mV. By how much could the transmitter power be reduced if the DC component were reduced to its optimum value? [0.12 dB]

6.2. Find the mean and peak transmitter power required for a single section, 2 km, link which employs, unipolar RZ (50% mark space ratio), rectangular pulse signalling and centre point detection if the probability of bit error is not to exceed 10^{-6}. The specific attenuation of the (perfectly equalised) link transmission line is 1.8 dB/km and the (Gaussian) noise power at the receiver decision circuit input is 1.8 mW. (Assume that the (real) impedance level at transmitter output and decision circuit input are both equal to the characteristic impedance of the transmission line.) [93 mW, 0.37 W]

6.3. A 1.0 Mbaud, baseband, digital signal has 8 allowed voltage levels which are equally spaced between +3.5 V and −3.5 V. If this signal is transmitted over a 26 section, regenerative repeater link, and assuming that all the sections of the link are identical, find the RMS noise voltage which could be tolerated at the end of each hop whilst maintaining an overall link symbol error rate of 1 error/s. [91.4 mV]

6.4. What are the two factors which control the power spectral density (PSD) of a line coded communications signal? Using the fact that the Fourier transform of a rectangular pulse has a characteristic $(\sin x)/x$ shape, approximate the PSD and consequent channel bandwidth requirements for a return to zero, on-off keyed, line coded signal incorporating a positive pulse whose width equals one-third the symbol interval. Compare this with a non-return to zero signal, sketching the line coded waveforms and PSDs for both cases.

How do these figures compare with the minimum theoretical transmission bandwidth? What are the practical disadvantages of the on–off keyed waveform?

6.5. A binary transmission scheme with equiprobable symbols uses the absence of a voltage pulse to represent a digital 0 and the presence of the pulse $p(t)$ to represent a digital 1 where $p(t)$ is given by:

$$p(t) = \Pi\{t/(T_o/3)\} - 0.5\,\Pi\{[t - (T_o/3)]/(T_o/3)\} - 0.5\,\Pi\{[t + (T_o/3)]/(T_o/3)\}$$

Sketch the pulse, $p(t)$, and comment on the following aspects of this line code: (a) its DC level; (b) its suitability for transmission using AC coupled repeaters; (c) its (first null) bandwidth; (d) its P_e performance, using ideal centre point decision, compared with unipolar NRZ rectangular pulse signalling; (e) its self clocking properties (referring to equation (6.21) if you wish); and (f) its error detection properties.

6.6. Justify the entries in the last (transparency) column of Table 6.1.

6.7. The power spectral density of a bipolar NRZ signal (Figure 6.11) is given by:

$$G(f) = V^2 T_o \,\text{sinc}^2(T_o f)\, \sin^2(2\pi T_o f)$$

Use this to help verify the power spectral density of the coded mark inversion (CMI) signal given in Figure 6.11. (A plot of your result is probably necessary to verify the PSD.) [Hint: since the correlation of the mark and space parts of the CMI signal is zero for all values of time shift, then the power spectra of these parts can be found separately and summed to get the power spectral density of the CMI signal.]

6.8. Draw a perfectly equalised prototype positive pulse for an AMI signal. Use it to construct the eye diagram for the signal +1, −1, +1, −1, + ⋯. Add and label the remaining trajectories for a random signal.

6.9. Explain why a non-linear process is required to recover clock timing from an HDB3 signal.

6.10. For Gaussian variables use the tables of erf(x) supplied in Appendix A to find:
(a) The probability that $x \leq 5.5$ if x is a Gaussian random variable with mean $\mu = 3$ and standard deviation $\sigma = 2$; (b) the probability that $x > 5.5$.
(c) Assuming the height of clouds above the ground at some location is a Gaussian random variable x with $\mu = 1830$ metres and $\sigma = 460$ metres, find the probability that clouds will be higher than 2750 metres.
(d) Find the probability that a Gaussian random variable with standard deviation of σ will exceed: (i) σ, (ii) 2σ, and (iii) 3σ. [(a) 0.11, (b) 0.89, (c) 0.023, (d) 0.159, 0.023, 0.0014]

6.11. Explain near end crosstalk. Provide a diagram to show the shape of the crosstalk spectrum at the regenerator output. How does this spectrum differ at the equaliser output.