CHAPTER 5

Sampling, multiplexing and PCM

5.1 Introduction

Chapter 1 provided an overview of a digital communications system and Chapters 2, 3 and 4 were reviews of some important concepts in the theories of signals, systems and noise. Chapter 5 is the starting point for a more detailed examination of the major functional blocks which make up a complete digital communications system. The discussion of these blocks will primarily be at systems level (i.e. it will concentrate on the inputs, outputs and functional relationships between them) rather than at the implementation level (the design of the electronic circuits which realise these relationships). Referring to Figure 1.3, the principal transmitter subsystems with which this chapter is concerned are the anti-aliasing filter, the sampling circuit, the quantiser, the PCM encoder and the baseband channel multiplexer. In the receiver we are concerned with the demultiplexer, PCM decoder and reconstruction filter. First, however, we give a brief review of pulse modulation techniques. This is mainly because pulse amplitude modulation (in particular) can be identified with the sampling operation preceding quantisation in Figure 1.3 and, as such, constitutes an important part of a digital communications transmitter. In addition, however, pulse modulations (generally) can be used as modulation schemes in their own right for analogue communications systems.

5.2 Pulse modulation

Pulse modulation describes the process whereby the amplitude, width or position of individual pulses in a periodic pulse train are varied (i.e. modulated) in sympathy with the amplitude of a baseband information signal, $g(t)$, Figure 5.1(a) to (d) (adapted from Stremler). Pulse modulation may be an end in itself allowing, for example, many separate information carrying signals to share a single physical channel by interleaving the individual signal pulses as illustrated in Figure 5.2 (adapted from Stremler). Such pulse interleaving is called time division multiplexing (TDM) and is discussed in detail in

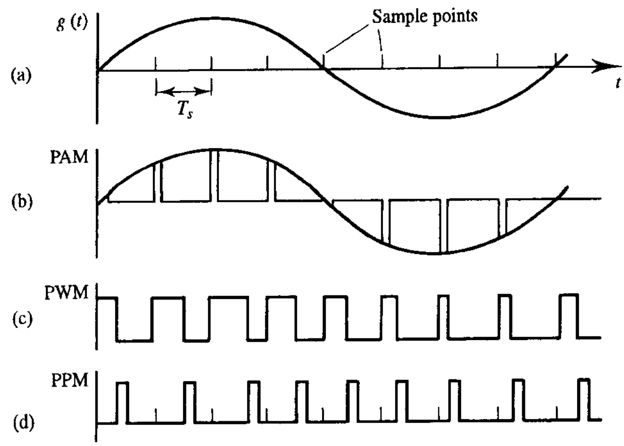


Figure 5.1 *Illustration of pulse amplitude, width and position modulation: (a) input signal.*

section 5.4. Pulse modulation also, however, represents an intermediate stage in the generation of digitally modulated signals. (It is important to realise that pulse modulation is not, in itself, a digital but an analogue technique.) The minimum pulse rate representing each information signal must be twice the highest frequency present in the signal's spectrum. This condition, called the Nyquist sampling criterion, must be satisfied if proper reconstruction of the original continuous signal from the pulses is to be possible. Sampling criteria, and the distortion (aliasing) which is introduced when they are not satisfied, are discussed further in section 5.3.

Since pulse amplitude modulation (PAM) relies on changes in pulse amplitude it requires larger signal-to-noise ratio (SNR) than pulse position modulation (PPM) or pulse width modulation (PWM) [Lathi]. This is essentially because a given amount of additive noise can change the amplitude of a pulse (with rapid rise and fall times) by a greater fraction than the position of its edges (Figure 5.3). PWM is particularly attractive in analogue remote control applications because the reconstructed control signal can easily be obtained by integrating (or averaging) the transmitted PWM signal.

All pulse modulated signals have wider bandwidth than the original information signal since their spectrum is determined solely by the pulse shape and duration. The bandwidth and filtering requirements for pulsed signals are discussed in Chapter 9. If the pulses are short compared with the reciprocal of the information signal bandwidth (or equivalently short with respect to the decorrelation time of the information signal) then the original continuous signal can be reconstructed by lowpass filtering. If the pulse duration is not sufficiently short then equalisation may be necessary after lowpass filtering. The need for, and effect of, equalisation are discussed in the context of sampling in section 5.3.1. Equalisation implementations are described in section 8.5.

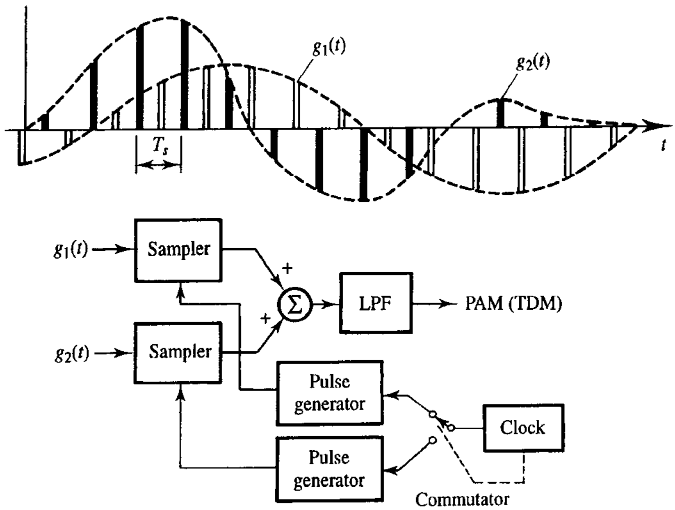


Figure 5.2 *Time division multiplexing of two pulse amplitude modulated signals.*

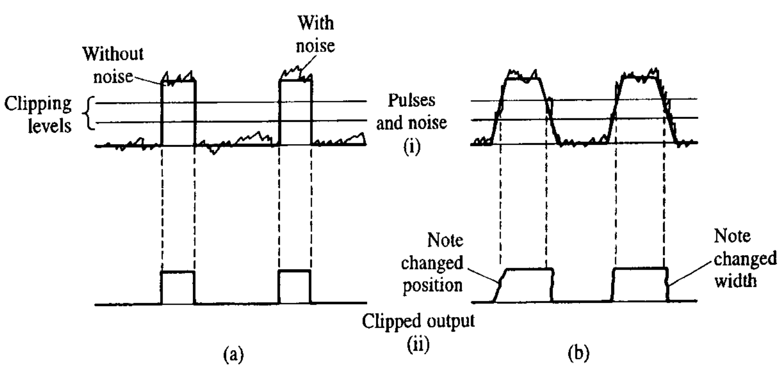


Figure 5.3 *Effects of noise on pulses: (a) noise induced position and width errors completely absent for ideal pulse; (b) small, noise induced, position and width errors for realistic pulse.*

5.3 Sampling

The process of selecting or recording the ordinate values of a continuous (usually analogue) function at specific (usually equally spaced) values of its abscissa is called sampling. If the function is a signal which varies with time then the samples are sometimes called a time series. This is the most common type of sampling process encountered in electronic communications although spatial sampling of images is also important.

There are obvious similarities between sampling and pulse amplitude modulation. In fact, in many cases, the two processes are indistinguishable, for instance if the pulse duration of the PAM signal is very short. There are, however, two processes both commonly referred to as sampling which should be distinguished. These are flat topped sampling (which is identical to PAM) and natural sampling.

The fundamental property of sampling is that, for a sampling frequency f_s a constant voltage DC input signal and a periodic input signal with a fundamental frequency at integer multiples of f_s both give (to within a multiplicative constant) the same sampled output values. A consequence of this, in the frequency domain, is that the sampled baseband spectrum repeats at f_s and multiples of this sampling frequency, Figure 5.4.

5.3.1 Natural and flat topped sampling

A naturally sampled signal is produced by multiplying the baseband information signal, $g(t)$, by the periodic pulse train, shown previously in Figure 2.36. This is illustrated in Figure 5.4(a), (c) and (e). The important point to note in this case is that the pulse tops follow the variations of the signal being sampled. The spectrum of the information signal is shown in Figure 5.4(b). (The spectrum is, of course, the Fourier transform of $g(t)$ and would normally be complex but is represented here only by its amplitude.) The spectrum

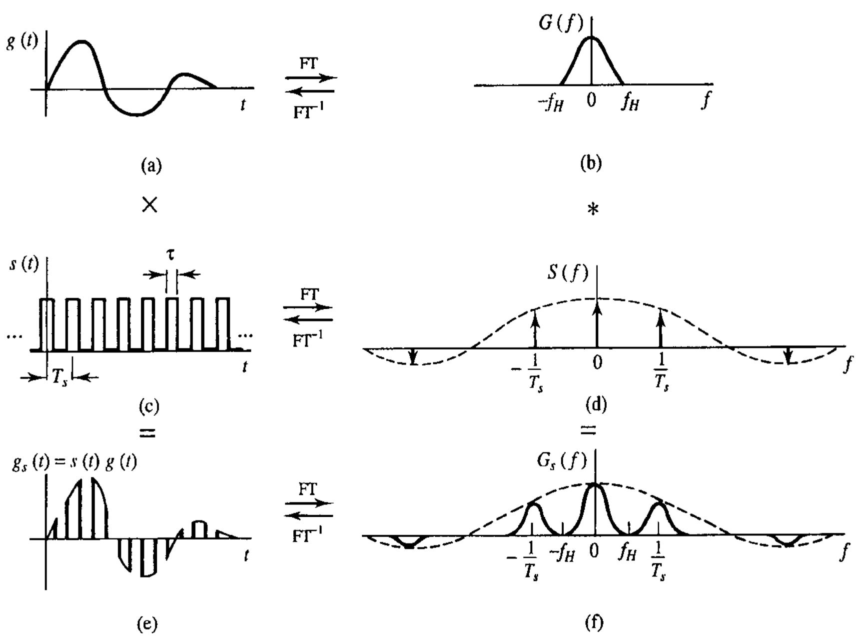


Figure 5.4 *Time and frequency domain illustrations of natural sampling: (a) signal $g(t)$; (b) signal spectrum; (c) sampling function; (d) spectrum of sampling function; (e) sampled signal; (f) spectrum of sampled signal.*

of the periodic pulse train (Figure 5.4(d)) is discrete and consists of a series of equally spaced, weighted, Dirac delta or impulse functions. The spacing between impulses is $1/T_s$ where T_s is the pulse train period and the envelope of weighted impulses is the Fourier transform of a single time domain pulse. (In this case the spectrum has a $\text{sinc}(\tau f)$ shape since the time domain pulses are rectangles of width τ seconds.) Since multiplication in the time domain corresponds to convolution in the frequency domain the spectrum of the sampled signal is found by convolving Figure 5.4(b) with Figure 5.4(d). It is a property of the impulse that, under convolution (section 2.3.4) with a second function, it replicates the second function about the position of the impulse. Each impulse in Figure 5.4 therefore replicates the spectrum of $g(t)$ at a frequency corresponding to its own position. The replicas have the same amplitude weightings as the impulses producing them. The sampled signal spectrum is shown in Figure 5.4(f). It is clear that appropriate lowpass filtering will pass only the baseband spectral version of $g(t)$. It follows that $g(t)$ will appear undistorted (and, in the absence of noise, without error) at the output of the lowpass filter. For obvious reasons such a filter is sometimes called a *reconstruction* filter.

If the pulses produced by the process described above are artificially flattened we have a true PAM signal or flat topped sampling. This can be modelled by assuming that natural sampling proceeds using an impulse train, Figure 5.5(a), (c), (e) (this is sometimes called impulse, or ideal, sampling) and the resulting time series of weighted impulses is convolved with a rectangular pulse, Figure 5.5(e), (g), (i). The resulting spectrum is that of the ideally sampled information signal (Figure 5.5(f)) multiplied with the $\text{sinc}(\tau f)$ spectrum of a single rectangular pulse, Figure 5.5(h), (j). A baseband spectral version of $g(t)$ can be recovered by lowpass filtering but this must then be multiplied by a function which is the inverse of the pulse spectrum ($1/\text{sinc}(\tau f)$, Figure 5.5(k)) if $g(t)$ is to be restored exactly. This process, which is not required for reconstruction of naturally sampled signals, is called equalisation. For proper equalisation the inverse frequency response in Figure 5.5(k) need only exist over the signal bandwidth, $0 \rightarrow f_H$.

5.3.2 Baseband sampling and Nyquist's criterion

The spectral replicas of the information signal in Figures 5.4(f) and 5.5(j) are spaced by $f_s = 1/T_s$ Hz. The baseband spectrum can therefore be recovered by simple lowpass filtering provided that the width of the spectral replicas is less than the spacing, as defined in equation (2.31), i.e.:

$$f_s \geq 2f_H \quad \text{(Hz)} \tag{5.1}$$

where f_H is the highest frequency component in the information signal. If, however, the spacing between spectral replicas is less than their width then they will overlap and reconstruction of $g(t)$ using lowpass filtering will no longer be possible. Equation (5.1) is a succinct statement of Nyquist's sampling criterion. In words this criterion could be stated as follows:

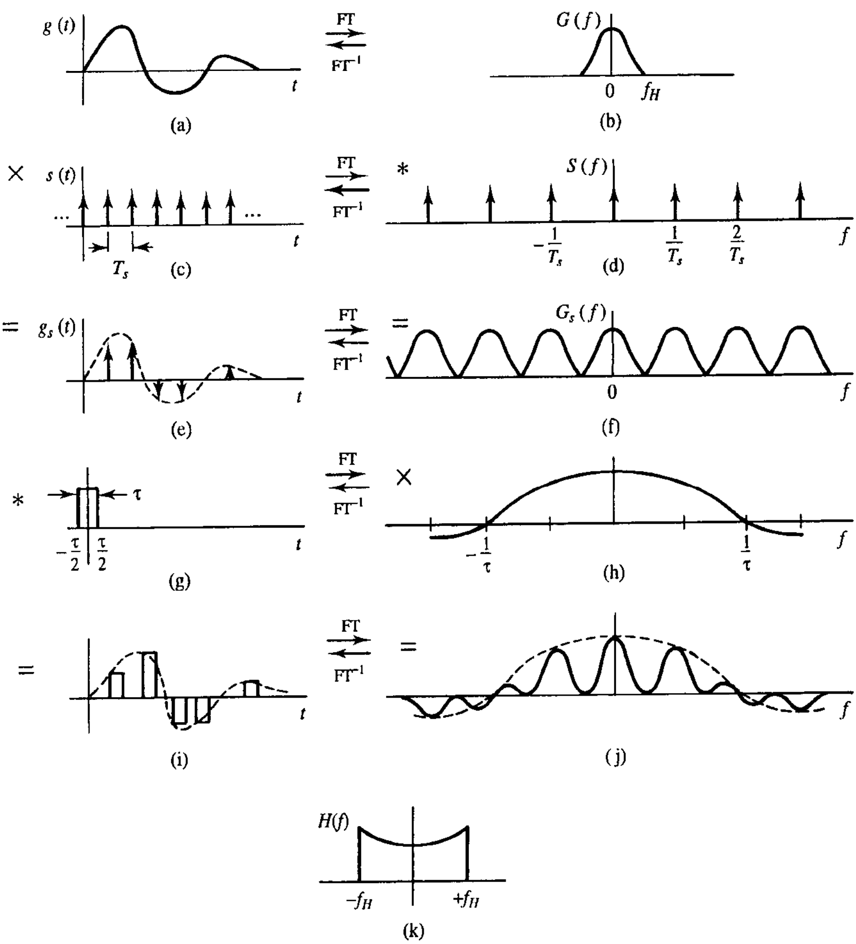


Figure 5.5 *Time and frequency domain illustrations of PAM or flat topped sampling: (a) signal; (b) signal spectrum; (c) sampling function; (d) spectrum of (c); (e) sampled signal; (f) spectrum of (e); (g) finite width sample; (h) spectrum of (g); (i) sampled signal; (j) spectrum of (i); (k) receiver equalising filter to recover $g(t)$.*

A signal having no significant spectral components above a frequency f_H Hz is specified completely by its values at uniform spacings, no more than $1/(2f_H)$ s apart.

Whilst this sampling criterion is valid for any signal it is usually only used in the context of baseband signals. Another less stringent (but more complicated) criterion which can be used for bandpass signals is discussed in section 5.3.5. In the context of sampling a strict distinction between baseband and bandpass signals can be made as follows:

For baseband signals, $B \geq f_L$ (Hz) (5.2(a))

For bandpass signals, $B < f_L$ (Hz) (5.2(b))

B in equations (5.2) is the signal's (absolute) bandwidth and f_L is the signal's lowest frequency component. These definitions are illustrated in Figure 5.6.

5.3.3 Aliasing

Figure 5.7 shows the spectrum of an undersampled baseband signal ($f_s < 2f_H$). The baseband spectrum of $g(t)$ clearly cannot be recovered exactly, even with an ideal rectangular lowpass filter. The best achievable, in terms of separating the baseband spectrum from the adjacent replicas, would be to use a rectangular lowpass filter with a cut-off frequency of $f_s/2$. The filtered signal will then be, approximately, that of the original signal $g(t)$ but with the frequencies above $f_s/2$ folded back so that they actually appear below $f_s/2$. (The approximation becomes better as the width of the sampling pulses gets smaller. In the limit of ideal (impulse) sampling the approximation becomes exact.) The spectral components originally representing high frequencies now appear under the alias of lower frequencies. Thus, as in Figure 2.23(c), the sampling represents a high frequency component by a lower frequency sinusoid.

To avoid aliasing a lowpass anti-aliasing filter with a cut-off frequency of $f_s/2$ is often placed immediately before the sampling circuit. Whilst this filter may remove high frequency energy from the information signal the resulting distortion is generally less

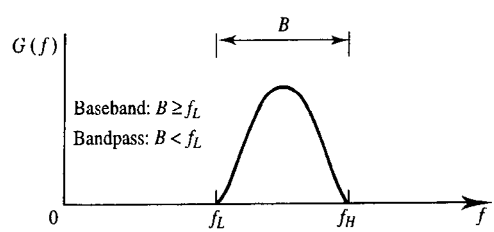


Figure 5.6 *Definitions of baseband and bandpass signals.*

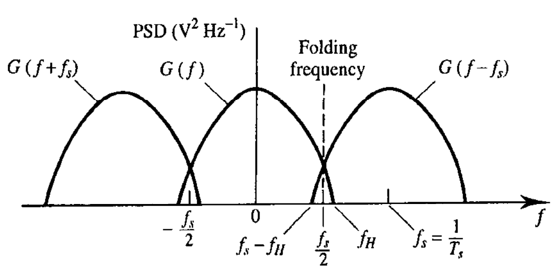


Figure 5.7 *Spectrum of undersampled information signal showing interference between spectral replicas and the folding frequency.*

than that introduced if the same energy is aliased to incorrect frequencies by the sampling process.

5.3.4 Practical sampling, reconstruction and signal to distortion ratio

Two practical points need to be appreciated when choosing the sampling rate in real systems. The first is that f_H must usually be interpreted as the highest frequency component *with significant spectral amplitude*. This is because practical signals start and stop in time and therefore, in principle, have spectra which are not bandlimited in an absolute sense. (For the case of voice signals, Figure 5.8, f_H, in Europe, is usually assumed to be 3.4 kHz.) The second is that whilst it is theoretically possible to reconstruct the original continuous signal from its samples if the sampling rate is exactly twice the highest frequency in its spectrum, in practice it is necessary to sample at a slightly faster rate. This is because ideal, rectangular, anti-aliasing and reconstruction filters (with infinitely steep skirts) are not physically realisable. A practical version of the baseband sampling criterion might therefore be expressed as:

$$f_s \geq 2.2 f_H \quad \text{(Hz)} \tag{5.3}$$

to allow for the transition, or roll-off, into the filter stopband.

A quantitative measure of the distortion introduced by aliasing can be defined as the ratio of unaliased to aliased power in the reconstructed signal. If the reconstruction filter is ideal with rectangular amplitude response then, in the absence of an anti-aliasing filter, the signal to distortion ratio (SDR) is given by:

$$\text{SDR} = \frac{\int_0^{f_s/2} G(f)\, df}{\int_{f_s/2}^{\infty} G(f)\, df} \tag{5.4}$$

where $G(f)$ is the (two sided) power spectral density of the (real) baseband information signal $g(t)$. More generally, if a filter with a frequency response $H(f)$ is used for reconstruction then the integral limits are extended (see Figure 5.7) to give:

$$\text{SDR} \approx \frac{\int_0^{\infty} G(f)\, |H(f)|^2\, df}{\int_0^{\infty} G(f - f_s)\, |H(f)|^2\, df} \tag{5.5}$$

An approximation sign is used in equation (5.5) for two reasons. Firstly, and most importantly, spectral replicas centred on $2f_s$ Hz and above are assumed to be totally suppressed by $|H(f)|$ in equation (5.5). Secondly, ideal (impulsive) sampling is assumed such that there is no $\text{sinc}(\tau f)$ roll-off in the spectrum of the sampled signal as discussed in section 5.3.1. (In the absence of $1/\text{sinc}(\tau f)$ equalisation then this additional spectral

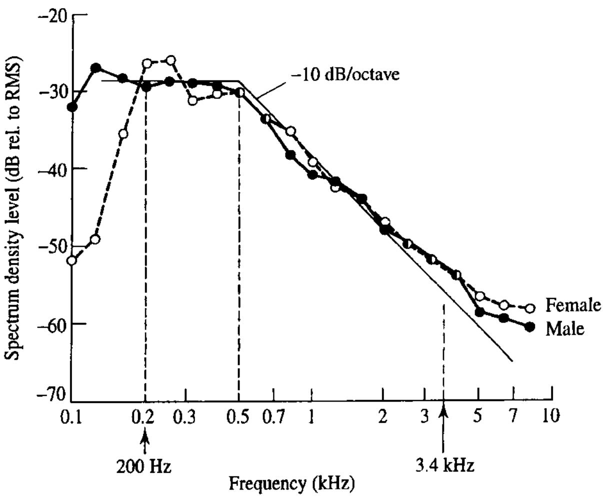


Figure 5.8 *Long-term averaged speech spectra for male and female speakers (source: Furui, 1989, reproduced with permission of Marcel Dekker).*

roll-off should be cascaded with $H(f)$ in equation (5.5) if flat topped sampling is employed.) It should also be appreciated that SDR in equations (5.4) and (5.5) is specifically defined to represent distortion due to aliasing of frequency components. Thus, for example, phase distortion due to the action of a reconstruction filter (with non-linear phase) on the baseband signal's voltage spectrum is not included.

EXAMPLE 5.1

Consider a signal with the power spectral density shown in Figure 5.9(a). Find the alias induced signal to distortion ratio if the signal is sampled at 90% of its Nyquist rate and the reconstruction filter has: (i) an ideal rectangular amplitude response; and (ii) an RC lowpass filter response with a 3 dB bandwidth of $f_s/2$.

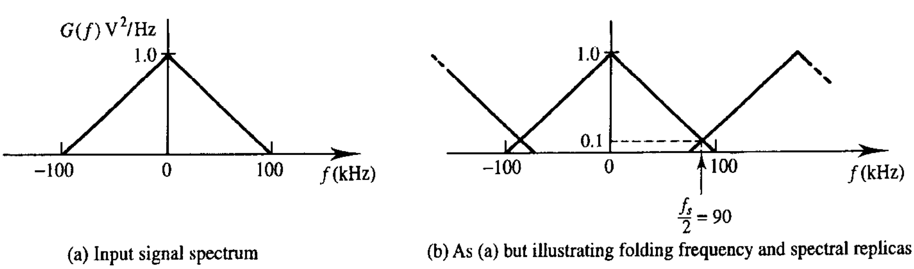


Figure 5.9 *Signal power spectral density for Example 5.1.*

The sampling rate is given by:

$$f_s = 0.9 \times 2\ f_H$$
$$= 0.9 \times 2 \times 100 = 180\ \text{kHz}$$

The folding frequency (Figure 5.9(b)) is thus:

$$\frac{f_s}{2} = 90\ \text{kHz}$$

(i) For an ideal rectangular reconstruction filter, using equation (5.4):

$$\text{SDR} = \frac{\int_0^{f_s/2} G(f)\,df}{\int_{f_s/2}^{\infty} G(f)\,df} = \frac{\int_0^{f_s/2} (1 - 10^{-5} f)\,df}{\int_{f_s/2}^{10^5} (1 - 10^{-5} f)\,df}$$

For such a simple $G(f)$, however, we can evaluate the integrals from the area under the triangles in Figure (5.9(b)), i.e.:

$$\text{SDR} = \frac{\tfrac{1}{2}(100 \times 1) - \tfrac{1}{2}(10 \times 0.1)}{\tfrac{1}{2}\ (10 \times 0.1)}$$
$$= 99 = 20.0\ \text{dB}$$

(ii) For the *RC* filter:

$$H(f) = \frac{1}{1 + j\,2\pi\ RC\ f}$$

$$|H(f)|^2 = \frac{1}{1 + (2\pi\ RC)^2\ f^2}$$

$$f_{3dB} = \frac{f_s}{2} = 90\ \text{kHz}$$

$$RC = \frac{1}{2\pi\ f_{3dB}} = \frac{1}{2\pi\ 9 \times 10^4} = 1.768 \times 10^{-6}\ \text{s}$$

Reducing the upper limit in the numerator of equation (5.5) to f_H Hz, as this is the maximum signal frequency in the baseband spectrum, and replacing the limits in the denominator with $f_s - f_H$ and $f_s + f_H$ since these are lower and upper frequency limits in the first spectral replica:

$$\text{SDR} = \frac{\int_0^{f_H} (1 - 10^{-5} f)\,[1 + (2\pi\ 1.768 \times 10^{-6})^2 f^2]^{-1}\,df}{\int_{f_s - f_H}^{f_s + f_H} [1 - 10^{-5}\,|f - f_s|]\,[1 + (2\pi\ 1.768 \times 10^{-6})^2 f^2]^{-1}\,df}$$

Evaluating the numerator:

$$\text{Num} = \int_0^{10^5} \frac{1 - 10^{-5} f}{1 + a^2 f^2}\,df$$

(where $a = 2\pi\ 1.768 \times 10^{-6} = 1.111 \times 10^{-5}$)

$$= \int_0^{10^5} \frac{1}{1 + a^2 f^2}\, df - 10^{-5} \int_0^{10^5} \frac{f}{1 + a^2 f^2}\, df$$

Put $x = af$, $\dfrac{dx}{df} = a$; when $f = 0$, $x = 0$ and when $f = 10^5$, $x = 1.111$

$$\text{Num} = \int_0^{1.111} \frac{1}{1 + x^2} \frac{dx}{a} - 10^{-5} \int_0^{1.111} \frac{1}{a} \frac{x}{1 + x^2} \frac{dx}{a}$$

$$= \frac{1}{a}\Big[\tan^{-1} x\Big]_0^{1.111} - \frac{10^{-5}}{a^2}\left[\frac{1}{2}\ln(1 + x^2)\right]_0^{1.111}$$

$$= 7.542 \times 10^4 - 3.256 \times 10^4 = 4.286 \times 10^4$$

Evaluating the denominator:

$$\text{Denom} = \int_{f_s - f_H}^{f_s} \frac{1 - 10^{-5} f_s + 10^{-5} f}{1 + a^2 f^2}\, df + \int_{f_s}^{f_s + f_H} \frac{1 + 10^{-5} f_s - 10^{-5} f}{1 + a^2 f^2}\, df$$

$$= (1 - 10^{-5} \times 180 \times 10^3) \int_{80 \times 10^3}^{180 \times 10^3} \frac{1}{1 + a^2 f^2}\, df + 10^{-5} \int_{80 \times 10^3}^{180 \times 10^3} \frac{f}{1 + a^2 f^2}\, df$$

$$+ (1 + 10^{-5} \times 180 \times 10^3) \int_{180 \times 10^3}^{280 \times 10^3} \frac{1}{1 + a^2 f^2}\, df - 10^{-5} \int_{180 \times 10^3}^{280 \times 10^3} \frac{f}{1 + a^2 f^2}\, df$$

Using $x = af$, when $f = 80 \times 10^3$, $x = 0.8888$, when $f = 180 \times 10^3$, $x = 2.000$, and when $f = 280 \times 10^3$, $x = 3.111$

$$\text{Denom} = -0.8\, \frac{1}{a}\Big[\tan^{-1}(x)\Big]_{0.8888}^{2.0} + 10^{-5} \frac{1}{a^2}\left[\frac{1}{2}\ln(1 + x^2)\right]_{0.8888}^{2.0}$$

$$+ 2.8\, \frac{1}{a}\Big[\tan^{-1}(x)\Big]_{2.0}^{3.111} - 10^{-5} \frac{1}{a^2}\left[\frac{1}{2}\ln(1 + x^2)\right]_{2.0}^{3.111}$$

$$= -7.201 \times 10^4\, [1.1071 - 0.7266] + \frac{8.102 \times 10^4}{2}\, [1.6094 - 0.5822]$$

$$+ 2.520 \times 10^5 [1.2598 - 1.1071] - \frac{8.102 \times 10^4}{2}\, [2.3682 - 1.6094]$$

$$= 2.195 \times 10^4$$

$$\text{SDR} = \frac{\text{Num}}{\text{Denom}} = \frac{4.286 \times 10^4}{2.195 \times 10^4} = 1.95 = 2.9\ \text{dB}$$

Comparing the results for (i) and (ii) shows that if significant aliased energy is present a good multipole filter with a steep skirt is essential for signal reconstruction if the SDR is to be kept to tolerable levels. Furthermore the SDR calculated in part (ii) is extremely optimistic since spectral replicas centered on $2f_s$ Hz and above have been ignored. For reconstruction filters with only gentle roll-off equation (5.5) should really be used only as an upper bound on SDR, see Problem 5.3.

5.3.5 Bandpass sampling

In some applications it is desirable to sample a bandpass signal in which the centre frequency is many times the signal bandwidth. Whilst, in principle, it would be possible to sample this signal at twice the highest frequency component in its spectrum and reconstruct the signal from its samples by lowpass filtering it is usually possible to retain all the information needed to reconstruct the original signal whilst sampling at a much lower rate. If advantage is taken of this then there exists one or more frequency *bands* in which the sampling frequency should lie. Thus when sampling a bandpass signal there is generally an upper limit for proper sampling as well as a lower limit. The bandpass sampling criterion can be expressed as follows:

A bandpass signal having no spectral components below f_L Hz or above f_H Hz is specified uniquely by its values at uniform intervals spaced $T_s = 1/f_s$ s apart provided that:

$$2B\left\{\frac{Q}{n}\right\} \leq f_s \leq 2B\left\{\frac{Q-1}{n-1}\right\} \tag{5.6}$$

where $B = f_H - f_L$, $Q = f_H/B$, n is a positive integer and $n \leq Q$.

The following comments are made to clarify the use of equation (5.6) and its relationship to the Nyquist baseband sampling criterion.

1. If $Q = f_H/B$ is an integer then $n \leq Q$ allows us to choose $n = Q$. In this case $f_s = 2B$ and the correct sampling frequency is exactly twice the signal bandwidth.
2. If $Q = f_H/B$ is not an integer then the lowest allowed sampling rate is given by choosing $n = int(Q)$ (i.e. the next lowest integer from Q). Lower values of n will still allow reconstruction of the original signal but the sampling rate will be unnecessarily high. (Lower values of n may, however, give a wider band of allowed f_s.)
3. If $Q < 2$ (i.e. $f_H < 2B$ or, equivalently, $f_L < B$) then $n \leq Q$ means that $n = 1$.
   In this case:

   $$2BQ \leq f_s \leq \infty \quad \text{(Hz)}$$

   and since $BQ = f_H$ we have:

   $$2f_H \leq f_s \leq \infty \quad \text{(Hz)}$$

   This is a statement of the Nyquist (baseband) sampling criterion.

The validity of the bandpass sampling criterion is most easily demonstrated using convolution (section 2.3.4) for the following special cases. (Convolution results in the sampled signal being replicated at DC and multiples of the sample frequency.)

1. When the spectrum of the bandpass signal $g(t)$ straddles nf_s, i.e. $G(f)$ straddles any of the lines in the spectrum of the sampling signal (Figure 5.10(a)), then convolution results in interference between the positive and negative frequency spectral replicas (Figure 5.10(b)).
2. When the spectrum of $g(t)$ straddles $(n+½)f_s$, i.e. $G(f)$ straddles any odd integer multiple of $f_s/2$ (Figure 5.10(c)), then similar interference occurs (Figure 10(d)).
3. When the spectrum of $g(t)$ straddles neither nf_s nor $(n+½)f_s$ (Figure 5.10(e)), then no interference between positive and negative frequency spectral replicas occurs (Figure 5.10(f)) and the baseband (or bandpass) spectrum can be obtained by filtering.

Summarising we have the following conditions for proper sampling:

$$f_H \leq n \frac{f_s}{2} \quad \text{(Hz)} \tag{5.7}$$

$$f_L \geq (n-1) \frac{f_s}{2} \quad \text{(Hz)} \tag{5.8}$$

Using $f_L = f_H - B$ we have:

$$\frac{2}{n} f_H \leq f_s \leq \frac{2}{n-1} (f_H - B) \quad \text{(Hz)} \tag{5.9}$$

Defining $Q = f_H/B$ gives the bandpass sampling criterion of equation (5.6).

EXAMPLE 5.2

The following examples illustrate the use and significance of equation (5.6). First consider a signal with centre frequency 9.5 kHz and bandwidth 1.0 kHz.

The highest and lowest frequency components in this signal are:

$$f_L = 9.0 \text{ kHz} \qquad f_H = 10.0 \text{ kHz}$$

Quotient Q is thus:

$$Q = f_H/B = 10.0/1.0 = 10.0$$

Applying the bandpass sampling criterion of equation (5.6):

$$2 \times 10^3 \left\{\frac{10}{n}\right\} \leq f_s \leq 2 \times 10^3 \left\{\frac{10-1}{n-1}\right\} \quad \text{(Hz)}$$

Since Q is an integer the lowest allowed sampling rate is given by choosing $n = Q = 10$, i.e.:

$$2.0 \leq f_s \leq 2.0 \text{ (kHz)}$$

The significant point here is that there is zero tolerance in the sampling rate if distortion is to be completely avoided. If n is chosen to be less than its maximum value, e.g. $n = 9$, then:

$$2.222 \leq f_s \leq 2.250 \text{ (kHz)}$$

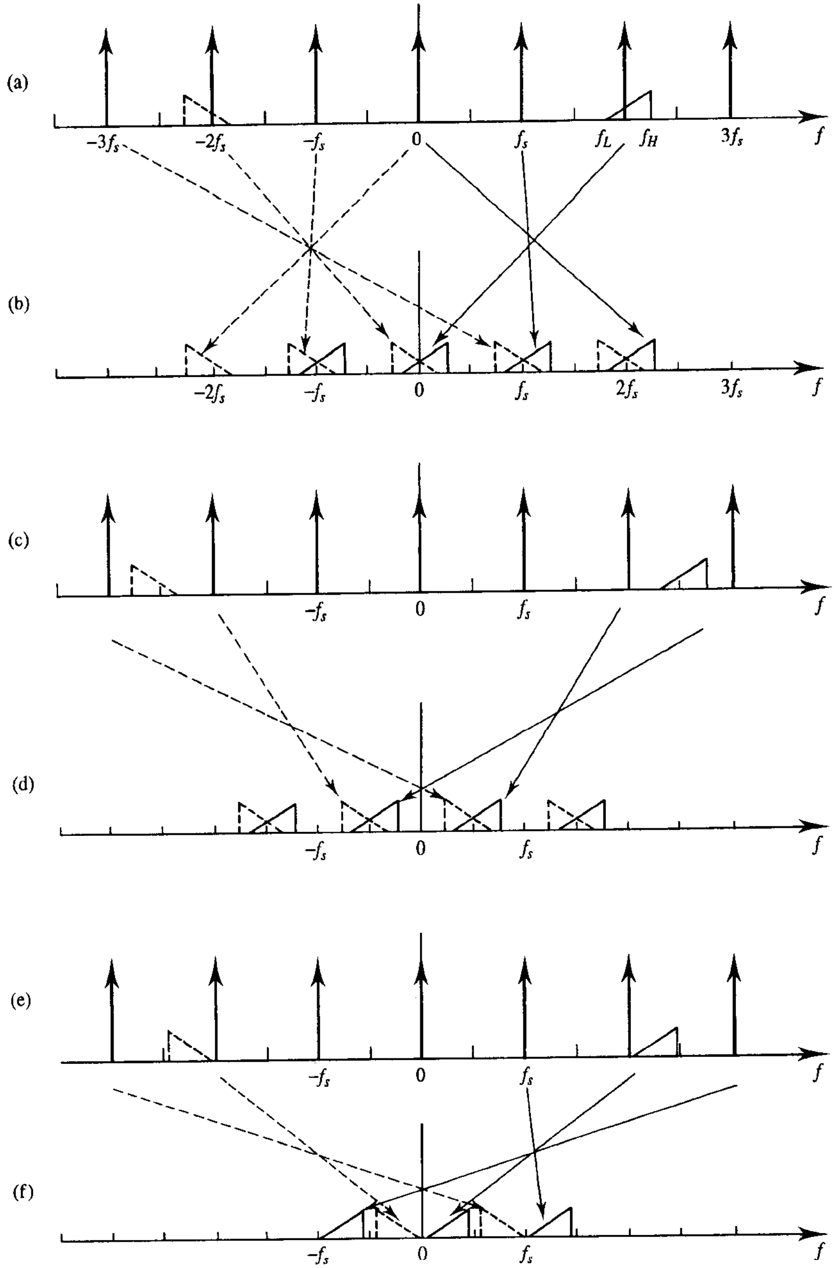


Figure 5.10 *Criteria for correct sampling of bandpass signals: (a) spectrum of signal where $G(f)$ straddles $2 \times f_s$; (c) spectrum of signal where $G(f)$ straddles $2.5 \times f_s$; (b) & (d) overlapped spectra after bandpass sampling; (e) bandpass spectrum avoiding the straddling in (a) & (c); (f) baseband or bandpass spectra, recoverable by filtering.*

The sampling rate in this case would be chosen to be 2.236 ± 0.014 kHz. The accuracy required of the sampling clock is therefore ± 0.63%. Now consider a signal with centre frequency 10.0 kHz and bandwidth 1.0 kHz. The quotient $Q = 10.5$ is now no longer an integer. The lowest allowed sampling rate is therefore given by $n = int(Q) = 10.0$. The sampling rate is now bound by:

$$2.100 \le f_s \le 2.111 \text{ (kHz)}$$

This gives a required sampling rate of 2.106 ± 0.006 kHz or 2.106 kHz ± 0.26%.

5.4 Analogue pulse multiplexing

In many communications applications different information signals must be transmitted over the same physical channel. The channel might, for example, be a single coaxial cable, an optical fibre, or, in the case of radio, the free space between two antennas. In order for the signals to be received independently (i.e. without cross-talk) they must be sufficiently separated in some sense.

This quality of separateness is usually called orthogonality, section 2.5.3. Orthogonal signals can be received independently of each other whilst non-orthogonal signals cannot. There are many ways in which orthogonality between signals can be provided. An intuitively obvious way, sometimes used in microwave radio communications, is to use two perpendicular polarisations for two independent information channels. Vertical and horizontal, linear, antenna polarisations are usually used but right and left hand circular polarisations may also be used. Such signals are orthogonal in polarisation.

The traditional way of providing orthogonality in analogue telephony and broadcast applications is to transmit different information signals using different carrier frequencies. Such signals (provided their spectra do not overlap) are disjoint in frequency and can be received separately using filters. Tuning the local oscillator in a superheterodyne receiver, Figure 1.4, also allows one signal to be separated from others with disjoint (and therefore orthogonal) frequency spectra. Using different carriers, or frequency bands, to isolate signals from each other in this way is called frequency division multiplexing (FDM).

In FDM telephony, 3.4 kHz bandwidth telephone signals, Figure 5.8, are stacked in frequency at 4 kHz spacings with small frequency guard bands between them to allow separation using practical filters. Figure 5.11 shows how an FDM signal can be generated and Figure 5.12 shows a schematic representation of an FDM signal spectrum generated using single (lower) sideband filters. FDM was the original multiplexing technique for analogue communications and is now experiencing a resurgence in fibre optic systems in which different wavelengths are used for simultaneous transmission of many information signals. In this particular context the term wavelength division multiplexing (WDM) is usually used in preference to FDM.

Orthogonality can be provided in a quite different way for pulse modulated signals. Instead of occupying separate frequency bands (as in FDM) the signals occupy separate time slots. This technique, illustrated in Figure 5.2, is called (analogue) time division

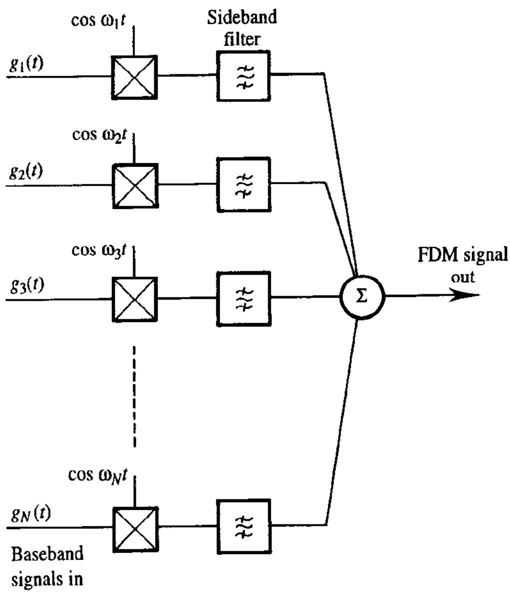


Figure 5.11 *Generation of an FDM signal.*

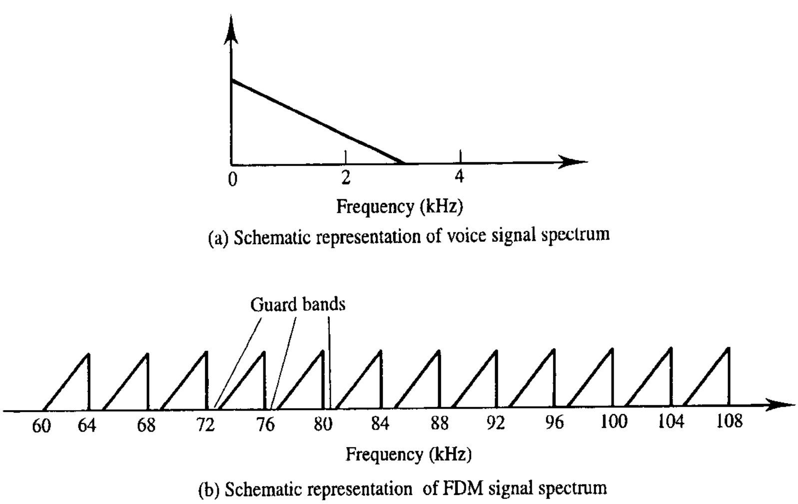


Figure 5.12 *FDM example for multiplexed speech channels.*

multiplexing (TDM). In telephony parlance the separate inputs in Figure 5.2 are called tributaries (see Chapter 19).

TDM obviously increases the overall sample rate and therefore the required bandwidth for transmission. It is, however, possible to reduce the bandwidth of the TDM

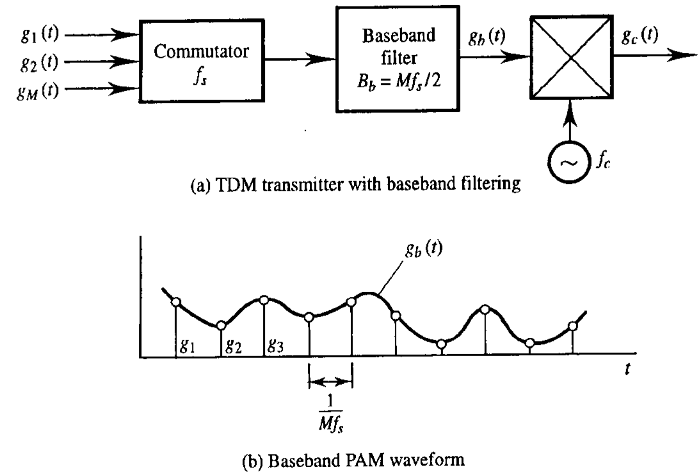


Figure 5.13 *Filtered TDM waveform.*

signal dramatically by appropriate filtering since, strictly, it is only necessary that the TDM signal provides the correct amplitude information at the sampling instants. Nyquist's sampling theorem says that a waveform with a bandwidth of $2f_H$ Hz exists which passes through these required points. This is illustrated in Figure 5.13. (At points between the sampling instants the filtered TDM signal is made up of a complicated sum of contributions arising from many pulses. At the sampling points themselves, however, the waveform amplitude is due to a single TDM pulse only.) If a minimum bandwidth TDM signal is formed by filtering then sampling accuracy becomes critical in that samples taken at times other than the correct instant will result in cross-talk between channels.

Cross-talk can also occur between the channels of a TDM signal even if filtering is not explicitly applied. This is because the transmission medium itself may bandlimit the signal. Such bandlimiting effects may often be at least approximated by RC low-pass filtering. In this case the response of the medium results in pulses with exponential rising and falling edges as shown in Figure 5.14. If the guard time between rectangular pulses (i.e. the time between the trailing edge of one pulse and the rising edge of the next) is t_g, and the time constant of the transmission channel is RC, then the amplitude of each pulse will decay to a fraction $e^{-t_g/RC}$ of its peak value by the time the next pulse starts. For the RC characteristic the channel bandwidth is $f_{3dB} = 1/(2\pi RC)$, therefore the cross-talk ratio (XTR) in dB at the pulse trailing edge (i.e. the optimum XTR sampling instant) is:

$$\begin{aligned} \text{XTR} &= 20 \log_{10} e^{2\pi f_{3dB}(t_g + \tau)} \\ &= 54.6\, f_{3dB}\,(t_g + \tau) \quad \text{(dB)} \end{aligned} \tag{5.10}$$

Equation (5.10) therefore gives a first order estimate of the required guard time to maintain a desired cross-talk ratio in a bandlimited channel, for a given rectangular pulse width, τ, at the channel input. (If sampling occurs at the centre of the τ s nominal pulse

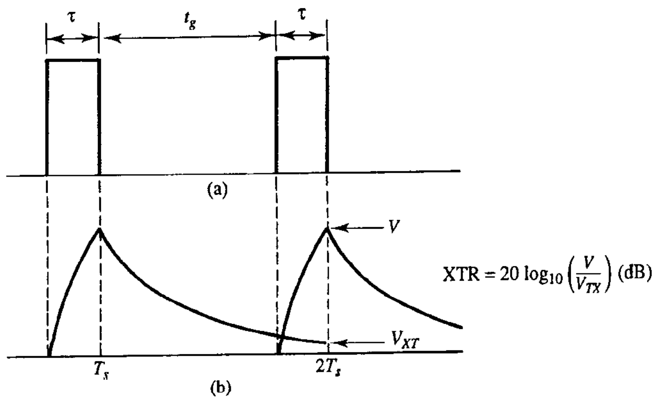


Figure 5.14 *Cross-talk between tributary channels of a TDM signal: (a) signal at channel input; (b) signal at channel output.*

slot, rather than at its end, then XTR is reduced by approximately 6 dB when $\tau \ll RC$.)

The discussion of TDM in this chapter has been mainly in the context of analogue pulse multiplexing. In Chapter 19 it is shown how (digital) TDM forms the basis of the telephone hierarchy for transmitting multiple simultaneous telephone calls over high speed 2, or 140, Mbit/s data links.

5.5 Quantised pulse amplitude modulation

An information signal which is pulse amplitude modulated becomes discrete (in time) rather than continuous but nevertheless remains analogue in nature since all pulse amplitudes within a specified range are allowed. An alternative way of expressing the analogue property of a PAM signal is to say that the probability density function (pdf) of pulse amplitudes is continuous (Figure 5.15). If a PAM signal is quantised, i.e. each pulse is adjusted in amplitude to coincide with the nearest of a finite set of allowed amplitudes (Figure 5.16) then the resulting signal is no longer analogue but digital and as a consequence has a discrete pdf as illustrated in Figure 5.17 (and previously in Figure 3.4).

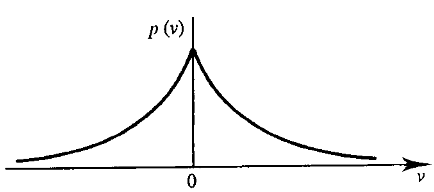
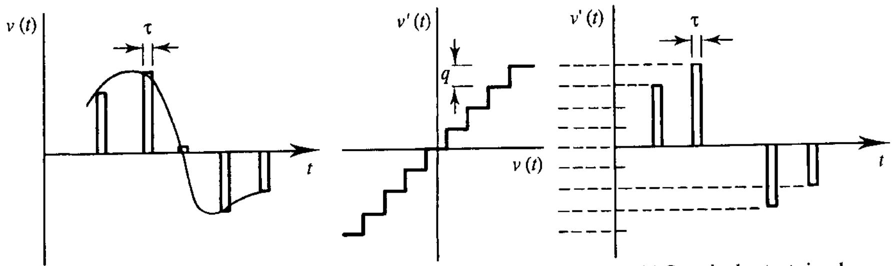


Figure 5.15 *Continuous pdf of typical analogue PAM signal.*



(a) Analogue input signal (b) Quantiser characteristics (c) Quantised output signal

Figure 5.16 *Quantisation of a PAM signal.*

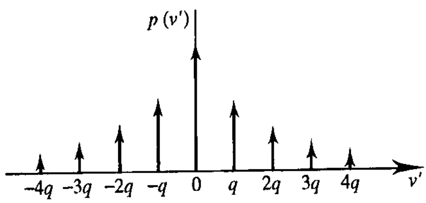


Figure 5.17 *Discrete pdf of quantised PAM signal.*

This digital signal can be represented by a finite set of symbols – the obvious set consisting of one symbol for each quantisation level. Other symbol sets (or alphabets) can be conceived, however, and unique mappings from one set to another established. Probably the simplest and most important alphabet is the binary set consisting of two symbols only, usually denoted by 0 and 1. The rest of this chapter is primarily concerned with the quantisation process and the subsequent efficient coding of quantised levels into binary symbols. In terms of Figure 1.3 the subsystems of principal importance here are the quantiser, pulse code modulation (PCM) encoder and source coder, although digital pulse multiplexing will also be discussed briefly. To some extent the separation of quantiser, PCM encoder and source coder might be misleading since they do not necessarily exist as identifiably separate pieces of hardware in all digital communications systems. For example, quantisation and PCM encoding are implemented together as a binary A/D converter (which may then be followed by a parallel to series converter) in many systems. Similarly some source coders (e.g. delta modulators) effectively *replace* the PCM encoder whilst others take a PCM signal and recode the binary symbols. In some systems (e.g. delta PCM) the source coder precedes the PCM encoder.

Quantising PAM signals is usually a precursor to generating pulse code modulation (PCM) which has some significant advantages over other baseband modulation types. The quantisation process in itself, however, actually degrades the quality of the information signal. This is easy to see since the quantised PAM signal no longer exactly represents the original, continuous analogue, signal but a distorted version of it. Figure 5.18 (which is drawn with PAM pulse width, τ, equal to the sampling period, T_s) shows that the quantised signal can be decomposed into the sum of the analogue signal and the difference between the quantised and the analogue signals. The difference signal is essentially random and can therefore be thought of as a special type of noise process. Like any other signal the power or RMS value of this *quantisation noise* can be calculated or measured. This leads to the concept of a *signal to quantisation noise ratio* (SN_qR).

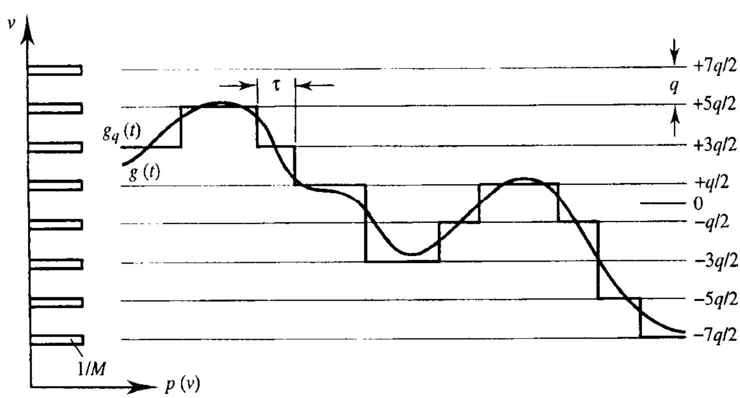
5.6 Signal to quantisation noise ratio (SN_qR)

To calculate the signal to quantisation noise ratio (SN_qR) of a quantised signal it is convenient to make the following assumptions:

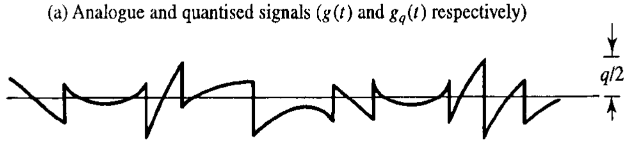
1. Linear quantisation (i.e. equal increments between quantisation levels).
2. Zero mean signal (i.e. symmetrical pdf about 0 V).
3. Uniform signal pdf (i.e. all signal levels equally likely).

The probability density function $p(v)$ of allowed levels is illustrated in the left hand side of Figure 5.18(a). (Narrow rectangles are used to represent the delta functions in the pdf to make interpretation easy.) Each rectangle has an area of $1/M$ where M is the (even) number of quantisation levels. (This is a result of assumption 3.) If the distance between adjacent quantisation levels is q V then the pdf of allowed levels is given by:

$$p(v) = \sum_{k=-M}^{M} (1/M)\,\delta(v - qk/2) \tag{5.11}$$



(a) Analogue and quantised signals ($g(t)$ and $g_q(t)$ respectively)



(b) Quantised minus analogue signal, $\varepsilon_q(t) = g_q(t) - g(t)$

Figure 5.18 *Quantisation error interpreted as noise, i.e.* $g_q(t) = g(t) + \varepsilon_q(t)$.

where k takes on odd values only. The mean square signal after quantisation is:

$$
\begin{aligned}
\overline{v^2} &= \int_{-\infty}^{+\infty} v^2 p(v)\, dv \\
&= \frac{2}{M}\left[\int_0^{\infty} v^2\, \delta(v - q/2)\, dv + \int_0^{\infty} v^2\, \delta(v - 3q/2)\, dv + \cdots\right] \\
&= \frac{2}{M}\left(\frac{q}{2}\right)^2 [1^2 + 3^2 + 5^2 + \cdots + (M-1)^2] \\
&= \frac{2}{M}\left(\frac{q}{2}\right)^2 \left[\frac{M(M-1)(M+1)}{6}\right]
\end{aligned}
\tag{5.12}
$$

i.e.:

$$\overline{v^2} = \frac{M^2 - 1}{12}\, q^2 \quad (V^2) \tag{5.13}$$

Denoting the quantisation error (i.e. the difference between the unquantised and quantised signals) as ε_q, Figure 5.18(b), then it follows from assumption 3 that the pdf of ε_q is uniform:

$$p(\varepsilon_q) = \begin{cases} 1/q, & -q/2 \le \varepsilon_q < q/2 \\ 0, & \text{elsewhere} \end{cases} \tag{5.14}$$

The mean square quantisation error (or noise) is:

$$\overline{\varepsilon_q^2} = \int_{-q/2}^{+q/2} \varepsilon_q^2 p(\varepsilon_q)\, d\varepsilon_q \tag{5.15}$$

i.e.:

$$\overline{\varepsilon_q^2} = \frac{q^2}{12} \quad (V^2) \tag{5.16}$$

The signal to quantisation noise ratio is therefore given by:

$$SN_qR = \overline{v^2}/\overline{\varepsilon_q^2} = M^2 - 1 \tag{5.17}$$

For large SN_qR the approximation $SN_qR = M^2$ is often used.

Equation (5.17) represents the average signal to quantisation noise (power) ratio. Since the *peak* signal level is $Mq/2$ V then the peak signal to quantisation noise (power) ratio is:

$$(SN_qR)_{peak} = \frac{(Mq/2)^2}{\overline{\varepsilon_q^2}} = 3M^2 \tag{5.18}$$

Expressed in dB the signal to quantisation noise ratios are:

$$\mathrm{SN_qR} = 20 \log_{10} M \text{ (dB)} \tag{5.19}$$

$$(\mathrm{SN_qR})_{peak} = 4.8 + \mathrm{SN_qR} \text{ (dB)} \tag{5.20}$$

5.7 Pulse code modulation

After a PAM signal has been quantised the possibility exits of transmitting not the pulse itself but a number indicating the height of the pulse. Usually (but not necessarily always) the pulse height is transmitted as a binary number. As an example, if the number of allowed quantisation levels were eight then the pulse amplitudes could be represented by the binary numbers from zero (000) to seven (111). The binary digits are normally represented by two voltage levels (e.g. 0 V and 5 V). Each binary number is called a code word and, since each quantised pulse is represented by a code word, the resulting modulation is called pulse code modulation (PCM). Figure 5.19(a) to (e) shows the relationship between an information, PAM, quantised PAM and PCM signal. Figure 5.19 (b) and (c) also illustrates the difference between a pulsed signal with duty cycle (τ/T_s) less than 1.0 and a pulsed signal with duty cycle equal to 1.0. The former are often referred to as return to zero (RZ) signals and the latter as non-return to zero (NRZ) signals.

There is clearly a bandwidth penalty to pay for PCM, if information is to be transmitted in real time, since, in the example given above, three binary pulses are transmitted instead of one quantised PAM pulse. (The penalty here is a factor of three since, for the same pulse duty cycle, each PCM pulse must be one third the duration of the PAM pulse.) The advantage of PCM is that for a given transmitted power the difference between adjacent voltage levels is much greater than for quantised PAM. This means that for a given RMS noise voltage the total voltage (signal plus noise) at the receiver is less likely to be interpreted as representing a level other than that which was transmitted. PCM signals are therefore said to have greater noise immunity than PAM signals.

5.7.1 $\mathrm{SN_qR}$ for linear PCM

Whilst it is true that PCM signals are more tolerant of noise than the equivalent quantised PAM signals it is also true that both suffer the same degradation due to quantisation noise. For a given number of quantisation levels, M, the number of binary digits required for each PCM code word is $n = \log_2 M$. The PCM peak signal to quantisation noise ratio, $(\mathrm{SN_qR})_{\mathrm{peak}}$, is therefore:

$$(\mathrm{SN_qR})_{peak} = 3M^2 = 3(2^n)^2 \tag{5.21}$$

If the ratio of peak to mean signal power, $v_{peak}^2/\overline{v^2}$, is denoted by α then the average signal to quantisation noise ratio is:

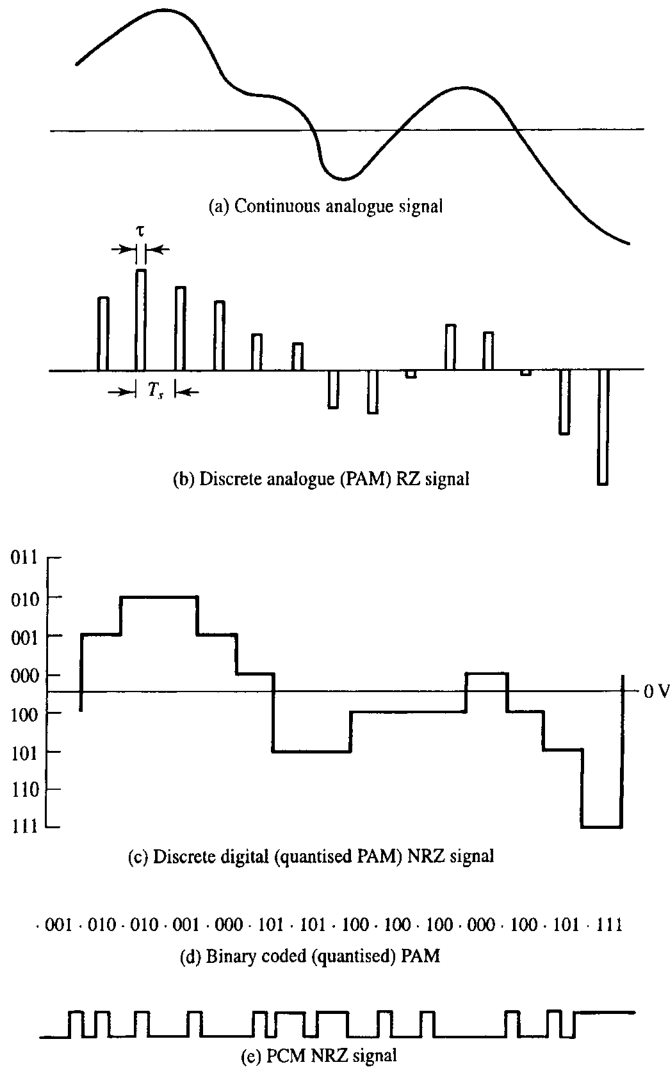


Figure 5.19 *Relationship between PAM, quantised PAM and PCM signal.*

$$\mathrm{SN_qR} = 3(2^{2n})\ (1/\alpha) \tag{5.22}$$

Expressed in dB this becomes:

$$\mathrm{SN_qR} = 4.8 + 6n - \alpha_{dB} \tag{5.23}$$

For a sinusoidal signal $\alpha = 2$ (or 3 dB). For a (clipped) Gaussianly distributed random signal (with $v_{peak}/\sigma_g = 4$ where σ_g is the signal's standard deviation, or RMS value, as in section 3.2.5) $\alpha = 16$ (or 12 dB), and for speech $\alpha = 10$ dB. The $\mathrm{SN_qR}$ for an n-bit PCM voice system can therefore be estimated using the rule of thumb $6(n-1)$ dB.

EXAMPLE 5.3
A digital communications system is to carry a single voice signal using linearly quantised PCM. What PCM bit rate will be required if an ideal anti-aliasing filter with a cut-off frequency of 3.4 kHz is used at the transmitter and the signal to quantisation noise ratio is to be kept above 50 dB?

From equation (5.23):

$$\text{SN}_\text{q}\text{R} = 4.8 + 6n - \alpha_{dB}$$

For voice signals $\alpha = 10$ dB, i.e.:

$$n = \frac{50 + 10 - 4.8}{6} = 9.2$$

10 bit/sample are therefore required. The sampling rate required is given by Nyquist's rule, $f_s \geq 2 f_H$. Taking a practical version of the sampling theorem, equation (5.3), gives:

$$f_s = 2.2 \times 3.4 \text{ kHz} = 7.48 \text{ kHz (or k samples/s)}$$

The PCM bit rate (or more strictly binary baud rate) is therefore:

$$\begin{aligned} R_b &= f_s n \\ &= 7.48 \times 10^3 \times 10 \text{ bit/s} \\ &= 74.8 \text{ kbit/s} \end{aligned}$$

5.7.2 SNR for decoded PCM

If all PCM code words are received and decoded without error then the SNR of the decoded signal is essentially equal to the signal to quantisation noise ratio, $\text{SN}_\text{q}\text{R}$, as given in equations (5.21) to (5.23). In the presence of channel and/or receiver noise, however, it is possible that one or more symbols in a given code word will be changed sufficiently in amplitude to be interpreted in error. For binary PCM this involves a digital 1 being interpreted as a 0 or a digital 0 being interpreted as a 1. The effect that such an error has on the SNR of the decoded signal depends on which symbol is detected in error. The least significant bit in a binary PCM word will introduce an error in the decoded signal equal to one quantisation level. The most significant bit would introduce an error of many quantisation levels.

The following reasonably simple analysis gives a useful expression for the SNR performance of a PCM system in the presence of noise.

We first assume that the probability of more than one error occurring in a single n-bit PCM code word is negligible. We also assume that all bits in the code word have the same probability (P_e) of being detected in error. Using subscripts $1, 2, \cdots, n$ to denote the significance of PCM code word bits (1 corresponding to the least significant, n corresponding to the most significant) then the possible errors in the decoded signal are:

$$\begin{aligned} \varepsilon_1 &= q \\ \varepsilon_2 &= 2q \\ \varepsilon_3 &= 4q \\ &\cdots \\ \varepsilon_n &= 2^{n-1}q \end{aligned} \tag{5.24}$$

The mean square decoding error, $\overline{\varepsilon_{de}^2}$ is the mean square of the possible errors multiplied by the probability of an error occurring in a code word, i.e.:

$$\begin{aligned} \overline{\varepsilon_{de}^2} &= nP_e(1/n)[(q)^2 + (2q)^2 + \cdots + (2^{n-1}q)^2] \\ &= P_e(q)^2[4^0 + 4 + 4^2 + 4^3 + \cdots + 4^{(n-1)}] \end{aligned} \tag{5.25}$$

The square bracket is the sum of a geometric progression with the form:

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1} \tag{5.26}$$

where $a = 1$ and $r = 4$. Thus:

$$\overline{\varepsilon_{de}^2} = P_e q^2(4^n - 1)/3 \ \ (\mathrm{V}^2) \tag{5.27}$$

Since the error or noise which results from incorrectly detected bits is statistically independent of the noise which results from the quantisation process we can add them together on a power basis, i.e.:

$$\mathrm{SNR} = \frac{\overline{v^2}}{\overline{\varepsilon_q^2} + \overline{\varepsilon_{de}^2}} \tag{5.28}$$

where $\overline{v^2}$ is the received signal power. Using equation (5.13) for $\overline{v^2}$ and equation (5.16) for $\overline{\varepsilon_q^2}$, and remembering that the number of quantisation levels $M = 2^n$ we have:

$$\mathrm{SNR} = \frac{M^2 - 1}{1 + 4(M^2 - 1)P_e} \tag{5.29}$$

Equation (5.29) allows us to calculate the average SNR of the decoded PCM signal including both quantisation noise and the decoding noise which occurs due to corruption of individual PCM bits by channel or receiver noise. In Chapter 6 expressions are developed which relate P_e to channel SNR. If we denote the channel SNR using the subscript *in* and the decoded PCM SNR using the subscript *out* then for binary, polar, NRZ signalling, using simple centre point decisions (see section 6.2), we have:

$$\mathrm{SNR}_{out} = \frac{M^2 - 1}{1 + 4(M^2 - 1)\ \tfrac{1}{2}\,\mathrm{erfc}\,(\tfrac{1}{2}\mathrm{SNR}_{in})^{\frac{1}{2}}} \tag{5.30}$$

The SNRs in equation (5.30) are linear ratios (not dB values) and the function erfc(x) is the complementary error function (defined later in equation (6.3)). Equation (5.30) is

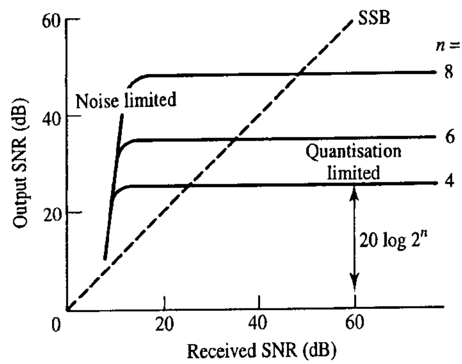


Figure 5.20 *Input/output SNR for PCM.*

sketched for various values of $n = \log_2 M$ in Figure 5.20. The noise immunity advantage of PCM illustrated by this figure is clear. The x-axis is the SNR of the received PCM signal. The y-axis is the SNR of the reconstructed (decoded) information signal. If the SNR of the received PCM signal is very large then the total noise is dominated by the quantisation process and the output SNR is limited to $\text{SN}_\text{q}\text{R}$. In practice, however, PCM systems are operated at lower input SNR values near the knee or threshold of the curves in Figure 5.20. The output SNR is then significantly greater than the input SNR. At very low input SNR, when the noise is of comparable amplitude to the PCM pulses, then the interpretation of code words starts to become unreliable. Since even a single error in a PCM code word can change its numerical value by a large amount then the output SNR in this region (i.e. below threshold) decreases very rapidly.

EXAMPLE 5.4

Find the overall SNR for the reconstructed analogue voice signal in Example 5.3 if receiver noise induces an error rate, on average, of one in every 10^6 PCM bits.

From equations (5.29) and (5.17):

$$\text{SNR}_{out} = \frac{\text{SN}_\text{q}\text{R}}{1 + 4\,\text{SN}_\text{q}\text{R}\,P_e}$$

$$\text{SN}_\text{q}\text{R} = 4.8 + 6n - \alpha_{dB} = 4.8 + (6 \times 10) - 10$$

$$= 54.8 \text{ dB (or } 3.020 \times 10^5)$$

$$\text{SNR}_{out} = \frac{3.020 \times 10^5}{1 + 4\,(3.020 \times 10^5)(1 \times 10^{-6})}$$

$$= 1.368 \times 10^5 = 51.4 \text{ dB}$$

The SNR (as a ratio) available with PCM systems increases with the square of the number of quantisation levels while the baud rate, and equivalently the bandwidth, increases with the logarithm of the number of quantisation levels. Thus bandwidth can be exchanged for SNR (as in analogue frequency modulation). Close to threshold PCM is superior to all analogue forms of pulse modulation (and it is also marginally superior to FM) at low SNR. However, all practical PCM systems have a performance which is an order of magnitude below their theoretical optimum.

As PCM signals contain no information in their pulse amplitude they can be regenerated using non-linear processing at each repeater in a long haul system. Such digital, regenerative, repeaters allow accumulated noise to be removed and essentially noiseless signals to be retransmitted to the next repeater in each section of the link. The probability of error does accumulate from hop to hop however. This is discussed in Chapter 6 (section 6.3).

5.7.3 Companded PCM

The expressions for SN_qR derived in section 5.6 (e.g. equations (5.19) and (5.20)) assume that the information signal has a uniform pdf, i.e. that all quantisation levels are used equally. For most signals this is not a valid assumption. If the pdf of the information signal is not uniform but is nevertheless known, and is constant with time, then it is intuitively obvious that to optimise the average SN_qR those quantisation levels used most should introduce least quantisation noise. One way to arrange for this to occur is to adopt non-linear quantisation or, equivalently, companding. Non-linear quantisation is illustrated in Figure 5.21(a). If the information signal pdf has small amplitude for a large fraction of time and large amplitude for a small fraction of time (as is usually the case) then the step size between adjacent quantisation levels is made small for low levels and larger for higher levels. Companding (*comp*ressing-exp*anding*) achieves the same result by compressing the information signal using a non-linear amplitude characteristic (Figure 5.22(a)) prior to linear quantisation and then expanding the reconstructed information signal with the inverse characteristic (Figure 5.22(b)). Ideally the companding characteristic would result in a signal which has a precisely uniform pdf. Whilst this ideal is unlikely to be achieved the compressed signal will have a *more nearly* uniform pdf and therefore better SN_qR than the uncompressed signal.

A rather different problem arises if the information signal has an unknown pdf or if its pdf (measured on some relevant time scale) changes with time. In the case of voice signals, for example, the pdf arising from an individual speaker is usually fairly constant and the *shape* of pdfs arising from different users are usually similar. (A typical voice exceedance curve is shown in Figure 5.23.) The gross signal level, however, can vary widely between speakers, with perhaps a man who habitually shouts whilst using the telephone at one extreme, and a woman who is especially softly spoken at the other extreme. In these cases the companding strategy is normally to maintain, as nearly as possible, a constant SN_qR for all signal levels. (This is quite a different strategy from maximising the average SN_qR.) Since quantisation noise power is proportional to q^2 then RMS quantisation noise voltage is proportional to q. If SN_qR is to be constant for all

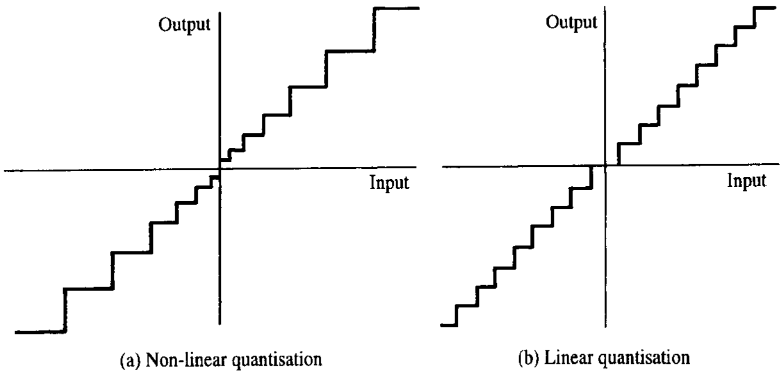


Figure 5.21 *Quantisation characteristics.*

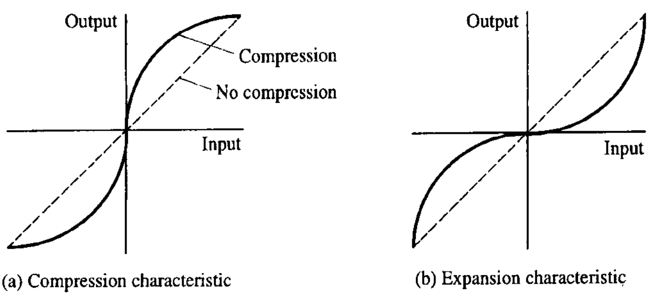


Figure 5.22 *Typical compression and expansion (compander) characteristics.*

signal levels then q must clearly be proportional to signal level, i.e. v/q must be a constant. If uniform quantisation is used then the signal should be compressed such that increasing the input signal by a given *factor* increases the output signal by a corresponding *additional constant*. Thus equal quantisation increments in the output signal correspond to quantisation increments in the input signal which are equal *fractions* of the input signal. The function which converts multiplicative factors into additional constants is the logarithm and the constant SN_qR compression characteristic is therefore of the form $y = \log x$ (Figure 5.24(a)).

Since information signals can usually take on negative as well as positive values the logarithmic compression characteristic must be reflected to form an odd symmetric function (Figure 5.24(b)). Furthermore the characteristic must obviously be continuous across zero volts and so the two logarithmic functions are modified and joined by a linear section as shown in Figure 5.24(c). The actual compression characteristic used in Europe is the A-law defined by:

$$F(x) = \begin{cases} \text{sgn}(x)\dfrac{1+\ln(A|x|)}{1+\ln A}, & 1/A < |x| < 1 \\ \text{sgn}(x)\dfrac{A|x|}{1+\ln A}, & 0 < |x| < 1/A \end{cases} \tag{5.31}$$

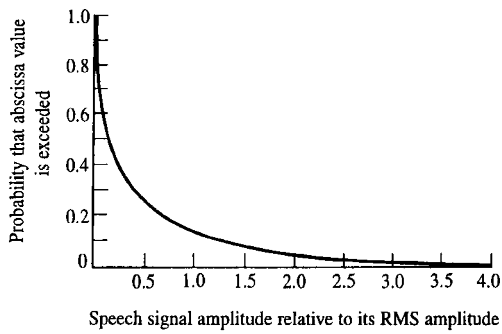


Figure 5.23 *Statistical distribution of single talker speech signal amplitude.*

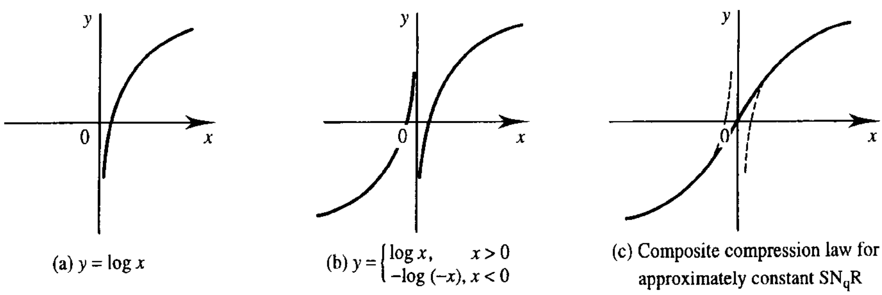


Figure 5.24 *Development of constant* SN_qR *compression characteristic.*

where $|x| = |v/v_{peak}|$ is the normalised input signal to the compressor, $F(x)$ is the normalised output signal from the compressor and $\text{sgn}(x)$ is the signum function which is $+1$ for $x > 0$ and -1 for $x < 0$.

The parameter A in equation (5.31) defines the curvature of the compression characteristic with $A = 1$ giving a linear law (Figure 5.25). The commonly adopted value is $A = 87.6$ which gives a 24 dB improvement in SN_qR over linear PCM for small signals ($|x| < 1/A$) and an (essentially) constant SN_qR of 38 dB for large signals ($|x| > 1/A$) [Dunlop and Smith]. The dynamic range of the logarithmic (constant SN_qR) region of this characteristic is $20\log_{10}[1/(1/A)] \approx 39$ dB. The overall effect is to allow 11 bit (2048 level) linear PCM, which would be required for adequate voice signal quality, to be reduced to 8 bit (256 level) companded PCM. A 4 kHz voice channel sampled at its Nyquist rate (i.e. 8 kHz) therefore yields a companded PCM bit rate of 64 kbit/s.

The A-law characteristic is normally implemented as a 13-segment piecewise linear approximation to equation (5.31) (in practice 16 segments but with 4 segments near the origin co-linear as illustrated in Figure 5.26). For 8 bit PCM one bit gives polarity, 3 bits indicate which segment the sample lies on and 4 bits provide the location on the segment.

In the USA and Japan a similar logarithmic compression law is used. This is the μ-law given by:

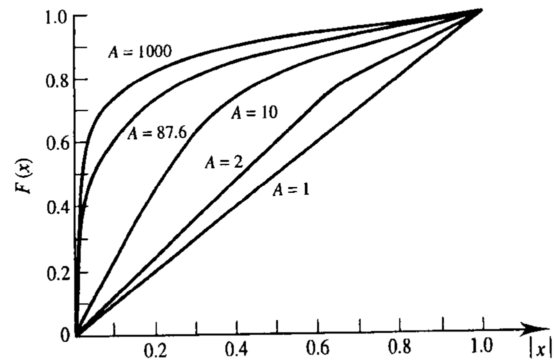


Figure 5.25 *A-law compression characteristic for several values of A.*

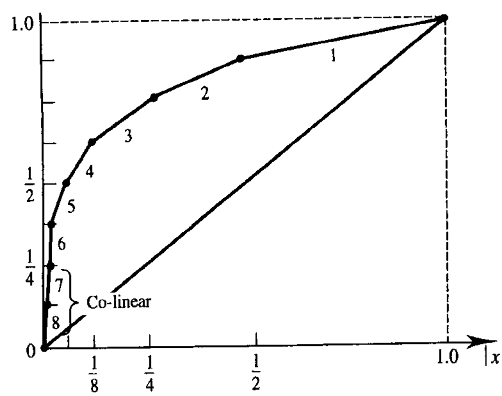


Figure 5.26 *13-segment compression A-law realised by piecewise linear approximation.*

$$F(x) = \mathrm{sgn}(x)\,\frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)}, \qquad 0 \le |x| \le 1 \tag{5.32}$$

The μ-law (with $\mu = 255$) tends to give slightly improved SN_qR for voice signals when compared with the A-law but it has a slightly smaller dynamic range. In practice, like the A-law, the μ-law is usually implemented as a piecewise linear approximation.

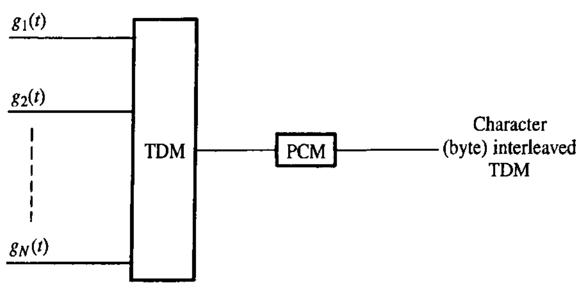
A- and μ-law 64 kbit/s companded PCM have been adopted by ITU-T as international toll quality standards (recommendation G.711) for digital coding of voice frequency signals. The sampling rate is 8 kHz and the encoding law uses 8 binary digits per sample. The A- and μ-laws are implemented as 13- and 15-segment piecewise linear curves respectively. For communications between countries using different companding laws (one using A and one using μ) conversion from one to the other is the responsibility of the country using the μ-law.

G.711 PCM communications has a *mean opinion score* (MOS) speech quality, measured by subjective testing, of 4.3 on a scale of 0 to 5. A MOS of 4 allows audible but not annoying degradation, 3 implies that the degradation is slightly annoying, 2 is annoying and 1 is very annoying. If narrow bandwidth is important then other coding techniques are employed (see section 5.8).

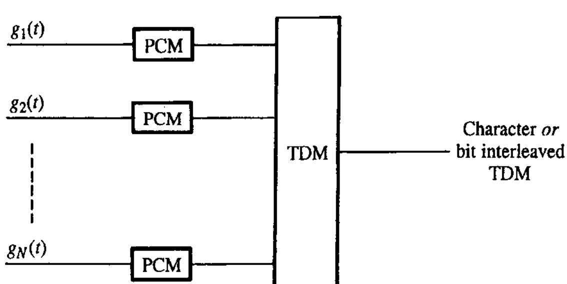
5.7.4 PCM multiplexing

Multiplexing of analogue PAM pulses has already been described in section 5.4. If such a multiplexed pulse stream is fed to a PCM encoder the resulting signal is a code word (or *character*) interleaved digital TDM signal. This is illustrated in Figure 5.27(a). In this figure the TDM signal is generated by *first* multiplexing and *then* PCM coding. It is of course possible to generate a character interleaved TDM signal by *first* PCM coding the information signals and *then* interleaving the code words, Figure 5.27(b). There are two reasons for drawing attention to this alternative method of TDM generation.

1. As digital technology has become cheaper and more reliable there has been a tendency for analogue signals to be digitally coded as near their source as possible.
2. Time division multiplexing of PCM signals (rather than PCM encoding of TDM PAM signals) suggests the possibility of interleaving the individual bits of the PCM code



(a) Analogue multiplexing followed by PCM



(b) PCM followed by digital multiplexing

Figure 5.27 *Generation of bit and byte interleaved PCM-TDM signals.*

words rather than the code word (bytes) themselves.

The TDM concept can be extended to higher levels of multiplexing by time division multiplexing two or more TDM signals. A detailed discussion of such multiplexing hierarchies is given in Chapter 19.

5.8 Bandwidth reduction techniques

Bandwidth is a limited, and therefore valuable, resource. This is because all physical transmission lines have characteristics which make them suitable for signalling only over a finite band of frequencies. (The problem of constrained bandwidth is especially severe for the local loop which connects an individual subscriber to their national telephone network.) Bandwidth (or more strictly spectrum) is even limited in the case of radio communications since the transmission properties of the earth's atmosphere are highly variable as function of frequency (see Chapter 14). Furthermore, co-channel (same frequency) radio transmissions are difficult to confine spatially and tend to interfere with each other, even when such systems are widely spaced geographically.

A given installation of transmission lines (wire-pairs, coaxial cables, optical fibres, microwave links and others) therefore represents a finite spectral resource and since adding to this installation (by laying new cables, for instance) is expensive, there is great advantage to be gained in using the existing installation efficiently. This is the real incentive to develop spectrally efficient (i.e. reduced bandwidth) signalling techniques for encoding speech signals.

5.8.1 Delta PCM

One technique to reduce the bandwidth of a PCM signal is to transmit information about the changes between samples instead of sending the sample values themselves. The simplest such system is delta PCM which transmits the difference between adjacent samples as conventional PCM code words. The difference between adjacent samples is, generally, significantly less than the actual sample values which allows the differences to be coded using fewer binary symbols per word than conventional PCM would require. (This reflects the fact that the adjacent samples derived from most, naturally generated, information signals are not usually independent but correlated, see section 2.6.) Block diagrams of a delta PCM transmitter and receiver are shown in Figure 5.28. It can be seen from this figure that the delta PCM transmitter simply represents an adjacent sample differencing operation followed by a conventional (usually reduced word length) pulse code modulator. Similarly the receiver is a pulse code demodulator followed by an adjacent 'sample' summing operation. The reduced number of bits per PCM code word translates directly into a saving of bandwidth. (This saving could, of course, then be traded against signal power and/or transmission time.) The delta PCM system cannot, however, accommodate rapidly varying transient signals as well as a conventional PCM system.

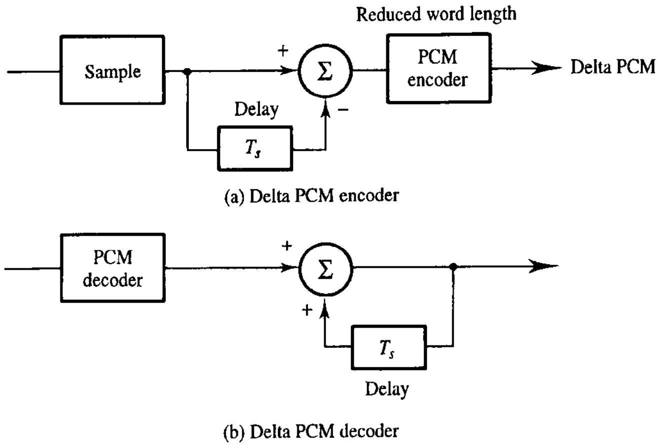


Figure 5.28 *Delta PCM transmitter and receiver.*

5.8.2 Differential PCM

The correlation between closely spaced samples of information signals originating from natural sources has already been referred to. An alternative way of expressing this phenomenon is to say that the signal contains redundant information, i.e. the same or similar information resides in two or more samples. An example of this is the redundancy present in a picture or image. To transmit an image over a digital communications link it is normally reduced to a two dimensional array of picture cells (pixels). These pixels are quantised in terms of colour and/or brightness. Nearly all naturally occurring images have very high average correlations, section 2.6, between the characteristics of adjacent pixels. Put crudely, if a given pixel is black then there is a high probability that the adjacent pixels will be at least nearly black. In this context the bandwidth of a random signal, the rate at which it is sampled, the correlation between closely spaced samples and its information redundancy are all closely related. One consequence of redundancy in a signal is that its future values can be predicted (within certain confidence limits) from its current and past values (see Chapter 16).

Differential PCM (DPCM) uses an algorithm to predict an information signal's future value. It then waits until the actual value is available for examination and subsequently transmits a signal which represents a correction to the predicted value. The correction signal is therefore a distillation of the information signal, i.e. it represents the information signal's surprising or unpredictable part. DPCM thus reduces the redundancy in a signal and allows the information contained in it to be transmitted using fewer symbols, less spectrum, shorter time and/or lower signal power. Figure 5.29(a) shows a block diagram of a DPCM transmitter. $g(t)$ is a continuous analogue information signal and $g(kT_s)$ is a sampled version of $g(t)$. k represents the (integer) sample number. $\varepsilon(kT_s)$ is the error between the actual value of $g(kT_s)$ and the value, $\hat{g}(kT_s)$, predicted from previous samples. It is this error which is quantised, to form $\varepsilon_q(kT_s)$, and encoded to give the

DPCM signal which is transmitted. $\tilde{g}(kT_s) = \hat{g}(kT_s) + \varepsilon_q(kT_s)$ is an estimate of $g(kT_s)$ and is the predicted value $\hat{g}(kT_s)$, *corrected* by the addition of the *quantised* error $\varepsilon_q(kT_s)$. From the output of the sampling circuit to the input of the encoder the system constitutes a type of source coder. The DPCM receiver is shown in Figure 5.29(b) and is identical to the predictor loop in the transmitter. The reconstructed signal at the receiver is therefore the estimate, $\tilde{g}(t)$, of the original signal $g(t)$.

The predictor in DPCM systems is often a linear weighted sum of previous samples (i.e. a transversal digital filter) implemented using shift registers [Mulgrew and Grant]. A schematic diagram of such a predictor is shown in Figure 5.30.

5.8.3 Adaptive DPCM

Adaptive DPCM (ADPCM) is a more sophisticated version of DPCM. In this scheme the predictor coefficients (i.e. the weighting factors applied to the shift register elements) are continuously modified (i.e. adapted) to suit the changing signal statistics. (The values of

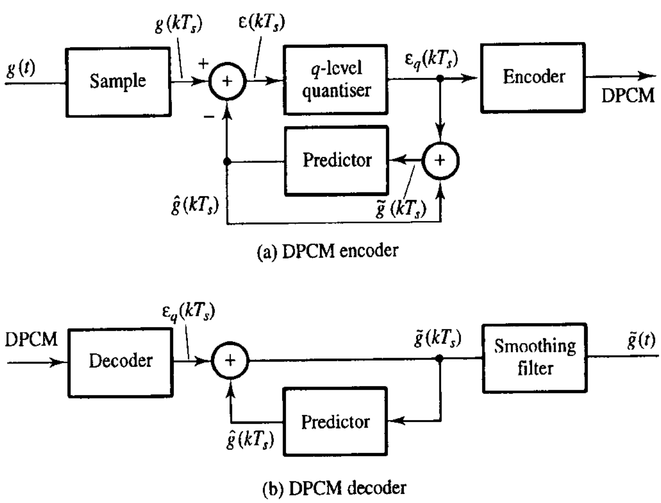


Figure 5.29 *DPCM transmitter and receiver.*

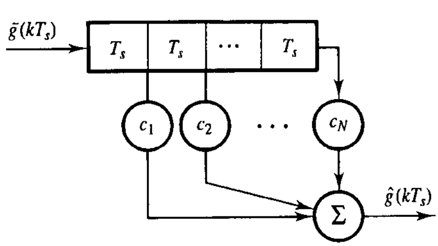


Figure 5.30 *DPCM predictor implemented as a transversal digital filter.*

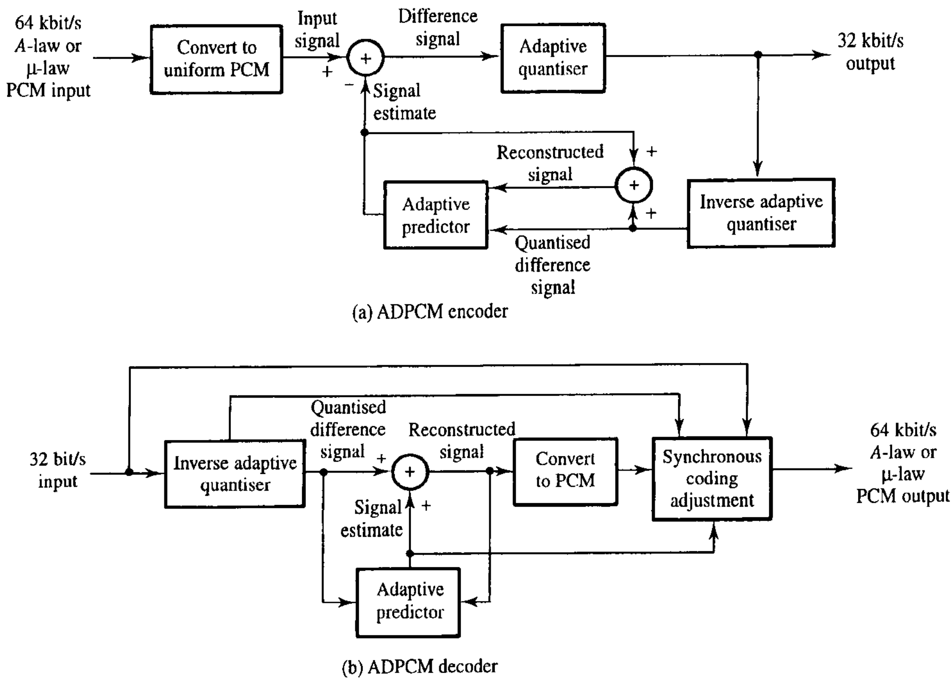
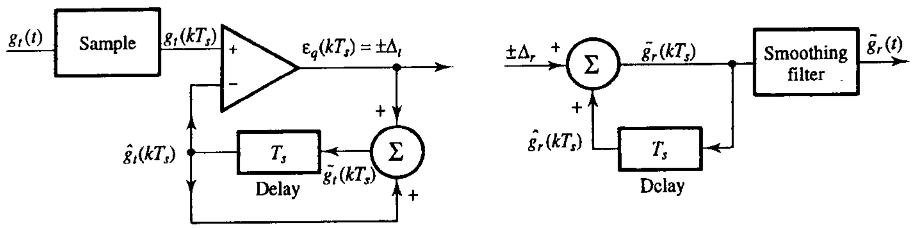


Figure 5.31 *ITU G.721 adaptive differential PCM (ADPCM) transmitter and receiver.*

these coefficients must, of course, also be transmitted over the communications link.) ITU has adopted ADPCM as a reduced bit rate standard. The ITU-T ADPCM encoder takes a 64 kbit/s companded PCM signal (G.711) and converts it to 32 kbit/s ADPCM signal (G.721). The G.721 encoder and decoder are shown in Figure 5.31. The encoder uses 15 level, 4-bit, codewords to transmit the quantised difference between its input and estimated signal. The subjective quality of error free 32 kbit/s ADPCM voice signals is only slightly inferior to 64 kbit/s companded PCM. For probabilities of error greater than 10^{-4} its subjective quality is actually better than 64 kbit/s PCM. 32 kbit/s ADPCM can achieve network quality speech with a MOS of 4.1 at error ratios of 10^{-3} to 10^{-2} for a complexity (measured by a logic gate count) of 10 times simple PCM. It allows an ITU-T 30 + 2 TDM channel (see Chapter 19) to carry twice the number of voice signals which are possible using 64 kbit/s companded PCM. Other specifications are defined by ITU-T, G.726 and G.727, for ADPCM with transmission rates of 16 to 40 kbit/s.

5.8.4 Delta modulation

If the quantiser of a DPCM system is restricted to one bit (i.e. two levels only, $\pm\Delta$) then the resulting scheme is called delta modulation (DM). This can be implemented by replacing the DPCM differencing block and quantiser with a comparator as shown in Figure 5.32(a). An especially simple prediction algorithm assumes that the next sample



(a) DM coder

(b) DM decoder

Figure 5.32 *DM transmitter (a) and receiver (b) using previous sample (or one tap) prediction.*

value will be the same as the last sample value, i.e. $\hat{g}(kT_s) = \tilde{g}[(k-1)T_s]$. This is called a previous sample predictor and is implemented as a one sample delay. The DM decoder is shown in Figure 5.32(b). Specimen signal waveforms at different points in the DM system are shown in Figure 5.33. Slope overload noise, which occurs when $g(t)$ changes too rapidly for $\tilde{g}(kT_s)$ to follow faithfully, and quantisation noise (also called granular noise) are illustrated in Figure 5.33. There is a potential conflict between the requirements for acceptable quantisation noise and acceptable slope overload noise. To reduce the former the step size Δ should be small and to reduce the latter Δ should be large. One way of keeping both types of noise within acceptable limits is to make Δ small, but sample much faster than the normal minimum (i.e. Nyquist) rate. Typically the DM sampling rate will be many times the Nyquist rate. (This does mean that the bandwidth saving which DM systems potentially offer may be partially or completely eroded.)

If the information signal $g(t)$ remains constant for a significant period of time then $\tilde{g}(kT_s)$ displays a hunting behaviour and the resulting quantisation noise becomes a square wave with a period twice that of the sampling period. When this occurs the quantisation noise is called idling noise. Since the fundamental frequency of this noise is half the sampling frequency, which is usually itself many times the highest frequency in the information signal, then much of the idling noise will be removed by the smoothing filter in the DM receiver.

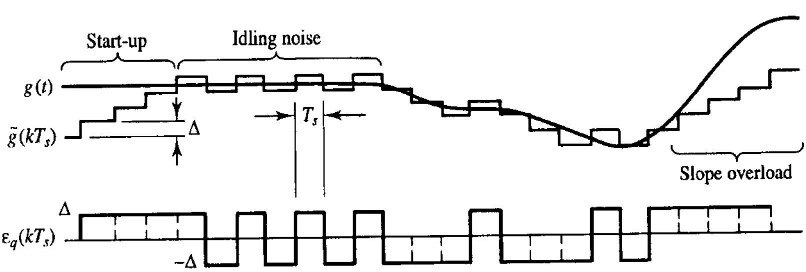


Figure 5.33 *DM signal waveforms illustrating slope overload and quantisation noise.*

A realistic analysis of the SNR performance of DM systems is rather complicated. Here we quote results adapted from [Schwartz, 1990] for a random information signal with Gaussian pdf and white spectrum bandlimited to f_H Hz. In this case the signal to slope overload noise ratio is:

$$\frac{S}{N_{ov}} = \frac{\sigma_g{}^2}{\sigma_{ov}{}^2} = 1.2\left(\frac{1}{2\pi}\frac{\Delta}{4\sigma_g}\frac{f_s}{f_H}\right)^5 e^{1.5\left(\frac{1}{2\pi}\frac{\Delta}{4\sigma_g}\frac{f_s}{f_H}\right)^2} \tag{5.33}$$

where σ_g is the standard deviation of the (Gaussian) information signal (equal to its RMS value if the DC component is zero), $N_{ov} = \sigma_{ov}^2$ is the variance (i.e. mean square) of the slope overload error, Δ is the DM step size, f_s is the DM sampling rate and f_H is the highest frequency in the baseband information signal (usually the information signal bandwidth). The signal to quantisation noise ratio (neglecting the effect of the smoothing filter) is:

$$\mathrm{SN_qR} = \frac{\sigma_g{}^2}{N_q} = 1.5\left(\frac{4\sigma_g}{\Delta}\right)^2 \frac{f_s}{f_H} \tag{5.34}$$

For a given peak signal level (assumed here to be $4\sigma_g$) the DM step size Δ can be reduced by a factor of 2 for each factor of 2 increase in f_s without introducing any more slope overload noise. DM $\mathrm{SN_qR}$ therefore potentially increases with f_s^3 leading to a $30\log_{10}2$ or 9 dB improvement for each octave increase in sampling frequency.

Assuming that the slope overload noise, N_{ov}, and quantisation noise, N_q, are statistically independent then they can be added power-wise to give the total signal to noise ratio, i.e.:

$$\mathrm{SNR} = \frac{S}{N_{ov} + N_q} = \frac{1}{(S/N_{ov})^{-1} + (S/N_q)^{-1}} \tag{5.35}$$

Equation (5.35) neglects the effects of channel and/or receiver noise which, if severe, might cause $+\Delta$ and $-\Delta$ symbols to be received in error. Figure 5.34 shows SNR plotted against the normalised step size – sampling frequency product, $(\Delta/4\sigma_g)(f_s/f_H)$, for various values of normalised sampling frequency, f_s/f_H. ($4\sigma_g$ is taken, for practical purposes, to be the maximum amplitude of the Gaussian information signal.) The peaky shape of the curves in Figure 5.34 reflects the fact that slope overload noise dominates for small DM step size and quantisation noise dominates for large step size.

If the SNR is not sufficiently high then the DM receiver will occasionally interpret a received symbol in error (i.e. $+\Delta$ instead of $-\Delta$ or the converse). This is equivalent to the addition of an error of 2Δ (i.e. the difference in the analogue signal represented by $+\Delta$ and $-\Delta$) to the accumulated signal at the DM receiver. The estimated signal $\tilde{g}_r(kT_s)$ at the receiver thereafter follows the variation of the estimated signal at the transmitter $\tilde{g}_t(kT_s)$ but with a constant offset of 2Δ. This situation continues until another error occurs which either cancels the first error or doubles it. The offset is therefore a stepped signal, step transitions occurring at random sample times with an average occurrence rate equal to the BER or P_e times the symbol rate. This is illustrated in Figure 5.35 which shows the result of errors being received in the DM signal and the consequent deviation from the

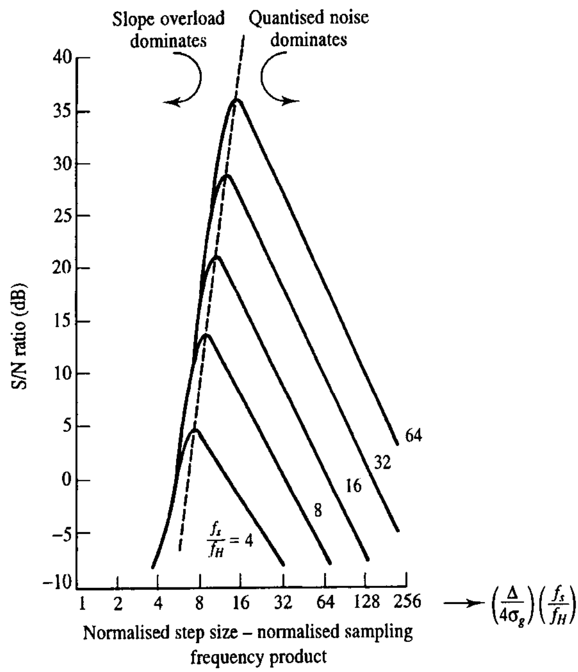


Figure 5.34 *SNR versus normalised step size - sampling frequency product for DM systems (parameter is normalised sampling frequency, (f_s/f_H)) (source: O'Neal, 1966, reproduced with permission of ATT Technical Journal).*

ideal output. On average the error (or noise) represented by such a signal increases without limit as errors accumulate. In practice, however, most of the power in this 'pseudo DC signal' is contained at low frequencies (assuming P_e is small). Provided the frequency response of the post accumulator (or smoothing) filter goes to zero at 0 Hz then most of this noise can be removed. A simple way of showing that this is the case is to consider a post accumulator filter which, close to 0 Hz, has a highpass RC characteristic. The response of this filter to a step change of 2Δ volts at its input is $2\Delta e^{-t/\tau_c}$ where the time constant $\tau_c = RC$. The energy, E_e, (dissipated in 1 Ω) at the filter output due to a single step transition at its input is:

$$E_e = \int_0^\infty [2\Delta\, e^{-t/\tau_c}]^2\, dt$$

$$= 2\Delta^2\tau_c \text{ (joules error}^{-1}) \tag{5.36}$$

The average 'noise' power, N_e, in the post filtered signal due to errors is therefore:

$$N_e = \text{BER} \times E_e \quad \text{(W)} \tag{5.37}$$

where BER is the bit (or more strictly symbol) error rate. Using BER = $P_e f_s$, where P_e

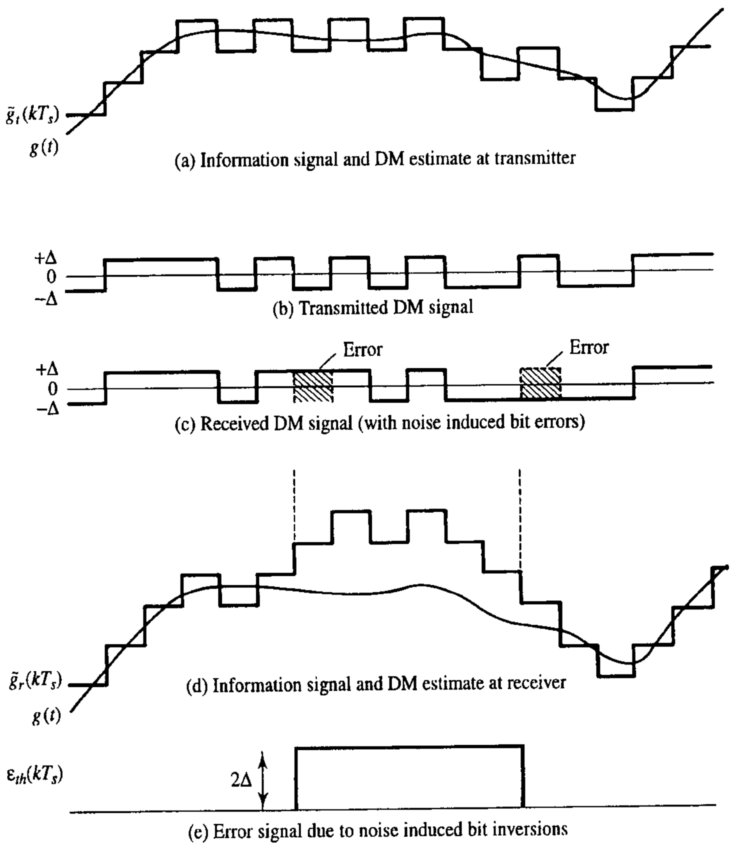


Figure 5.35 *Stepped error signal in DM receiver due to thermal noise.*

is the probability of error and f_s is the DM sampling rate, and putting $\tau_c = 1/2\pi f_L$ where f_L is the lowest frequency component with significant amplitude present in the information signal we have:

$$N_e = \frac{\Delta^2}{\pi} \frac{f_s}{f_L} P_e \tag{5.38}$$

Thus as f_L is increased more of the noise due to bit errors is removed. If an ideal rectangular highpass characteristic is assumed in place of the RC characteristic the result is smaller by a factor of $2/\pi$ (i.e. approximately 2 dB) [Taub and Schilling].

5.8.5 Adaptive delta modulation

In conventional DM the problem of keeping both quantisation noise and slope overload noise acceptably low is solved by oversampling, i.e. keeping the DM step size small and sampling at many times the Nyquist rate. The penalty incurred is the loss of some, or all, of the saving in bandwidth which might be expected with DM. An alternative strategy is to make the DM step size *variable*, making it larger during periods when slope overload noise would otherwise dominate and smaller when quantisation noise might dominate. Such systems are called adaptive DM systems (ADM).

A block diagram of an ADM transmitter is shown in Figure 5.36. The gain block, $G(kT_s)$, controls the variable step size represented by the *constant* amplitude pulses $\pm\Delta$. The step size is varied or adapted according to the history of ε_q. For example, if $\varepsilon_q = +\Delta$ for several adjacent samples it can be inferred that $g(t)$ is rising more rapidly than $\tilde{g}(kT_s)$ is capable of tracking it. Under this condition an ADM system increases the step size to reduce slope overload noise. Conversely if ε_q alternates between $+\Delta$ and $-\Delta$ then the inference is that $g(t)$ is changing slowly and slope overload is not occurring. The step size would therefore be decreased in order to reduce quantisation error. Figure 5.37 illustrates both these conditions. A simple ADM step size adjustment algorithm would be:

$$G(kT_s) = \begin{cases} G[(k-1)T_s]C, & \text{if } \varepsilon_q(kT_s) = \varepsilon_q[(k-1)T_s] \\ G[(k-1)T_s]/C, & \text{if } \varepsilon_q(kT_s) = -\,\varepsilon_q[(k-1)T_s] \end{cases} \tag{5.39}$$

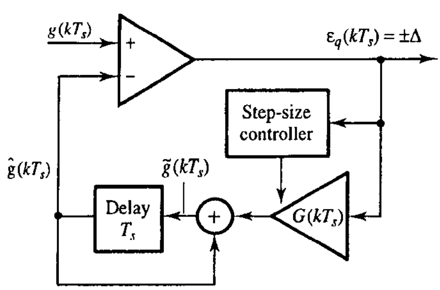


Figure 5.36 *Adaptive delta modulation (ADM) transmitter.*

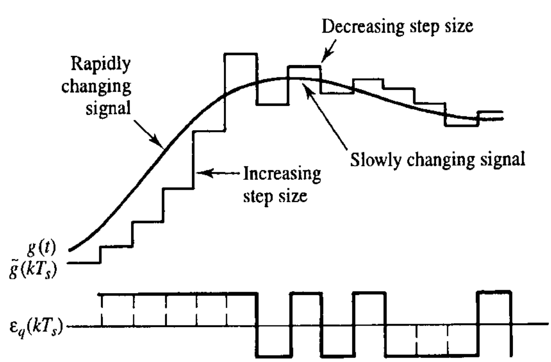


Figure 5.37 *ADM signal waveforms illustrating variable step size.*

where C is a constant > 1.

ADM typically performs with between 8 and 14 dB greater SNR than standard DM. Furthermore, since ADM uses large step sizes for wide variations in signal level and small step sizes for small variations in signal level it has wider dynamic range than standard DM. ADM can therefore operate at much lower bit rates than standard DM, typically 32 kbit/s and exceptionally 16 kbit/s. (16 kbit/s ADM allows reduced quality digital speech to be transmitted directly over radio channels which are allocated 25 kHz channel spacings.)

5.9 Summary

Continuous, analogue, information signals are converted to discrete, analogue, signals by the process of sampling. Pre-sampling anti-aliasing filters are used to limit any resulting distortion to acceptable levels. A naturally sampled signal can be converted to a PAM signal by flattening the pulse tops to give rectangular pulses. An ideal, impulse, sampled signal is similarly converted to a PAM signal by replacing the sample values with rectangular pulses of equivalent amplitude. PWM and PPM have a significant SNR advantage over PAM.

The minimum sampling rate required for an information signal with specified bandwidth is given by Nyquist's sampling theorem, if the signal is baseband, and by the bandpass sampling theorem if the signal is bandpass. In the bandpass case there is one or more allowed sampling rate bands rather than a simple sampling rate minimum. Reconstruction of a continuous information signal from a sampled, or PAM, signal can be achieved using a low-pass filter. In the absence of noise such reconstruction can be essentially error free. Practical filter designs usually mean that modest over-sampling is required if reconstruction is to be ideal.

Pulse modulated signals can be multiplexed together by interleaving the samples of the tributary information signals. The resulting TDM signal has narrower pulses, and therefore greater bandwidth, than is necessary for each of the tributary signals alone. It has the advantage, however, of allowing a single physical channel to carry many real time tributary signals, essentially simultaneously. Guard-times are normally required between the adjacent pulses in a TDM signal to keep cross-talk, generated by the bandlimited channel, to acceptable levels.

Quantisation, the process which converts an analogue signal to a digital signal, results in the addition of quantisation noise, which decreases as the number of quantisation levels increases. Pulse code modulation replaces M quantisation levels with M code words each comprising $n = \log_2 M$ binary digits or bits. A PCM signal then represents each binary digit by a voltage pulse which can have either of two possible amplitudes. The SN_qR of an n-bit, linear, PCM voice signal is approximately $6(n - 1)$ dB.

Companding of voice signals prior to PCM encoding increases the SN_qR of the decoded signal by increasing the resolution of the quantiser for small signals at the expense of the resolution for large signals. The ITU-T standard for digitally modulated companded voice signals is 8000 sample/s, 8-bit/sample giving a PCM bit rate of 64 kbit/s. Redundancy in transmitted PCM signals can be reduced by using DPCM techniques and its variants. ADPCM is an ITU standard for reduced bandwidth digital (32 kbit/s) voice transmission which gives received signal quality comparable to standard PCM. ADM, which is a variation on ADPCM, uses one bit quantisation.

5.10 Problems

5.1. (a) Sketch the design of TDM and FDM systems each catering for 12 voice channels of 4 kHz bandwidth.
(b) In the case of TDM, indicate how the information would be transmitted using: (i) PAM; (ii) PPM; (iii) PCM.
(c) Calculate the bandwidths required for the above FDM and TDM systems. You may assume the TDM system uses PPM with 2% resolution. [FDM 48 kHz; TDM 2.4 MHz]

5.2. Two lowpass signals, each band-limited to 4 kHz, are to be multiplexed into a single channel using pulse amplitude modulation. Each signal is impulse-sampled at a rate of 10 kHz. If the time-multiplexed signal waveform is filtered by an ideal lowpass filter (LPF) before transmission:
(a) What is the minimum clock frequency (or baud rate) of the system? [20 kHz]
(b) What is the minimum cut-off frequency of the LPF? [10 kHz]

5.3. Rewrite equation (5.5) such that it takes all spectral replicas into account. Hence find the, aliasing induced, SDR for Example 5.1 (part (ii)) accounting for the spectral replicas centred on f_s and $2f_s$ Hz.

5.4. An analogue bandpass signal has a bandwidth of 40 kHz and a centre frequency of 10 MHz. What is the minimum theoretical sampling rate which will avoid aliasing? What would be the best practical choice for the nominal sampling rate? Would an oscillator with a frequency stability of 1 part in 10^6 be adequate for use as the sampling clock? [80.16 kHz, 80.1603 kHz]

5.5. A rectangular PAM signal, with pulse widths of 0.1 μs, is transmitted over a channel which can be modelled by an *RC* low-pass filter with a half power bandwidth of 1.0 MHz. If an average XTR of 25 dB or better is to be maintained, estimate the required guard time between pulses. (Assume that pulse sampling at the channel output occurs at the optimum instants.) [0.36 μs]

5.6. Explain why the XTR in Problem 5.5 is degraded if the pulses are sampled (prematurely) at the mid point of the nominal, rectangular pulse, time slots. Quantify this degradation for this particular system. [7.5 dB]

5.7. Twenty-five input signals, each band-limited to 3.3 kHz, are each sampled at an 8 kHz rate then time-multiplexed. Calculate the minimum bandwidth required to transmit this multiplexed signal in the presence of noise if the pulse modulation used is: (a) PAM; (b) quantised PPM with a required level resolution of 5%; or (c) binary PCM with a required level resolution of > 0.5%. (This higher resolution requirement on PCM is normal for speech-type signals because the quantisation noise is quite objectionable.) [(a) 100 kHz, (b) 2 MHz, (c) 800 kHz]

5.8. A hi-fi music signal has a bandwidth of 20 kHz. Calculate the bit rate required to transmit this as a, linearly quantised, PCM signal maintaining a SN_qR of 55 dB. What is the minimum (baseband) bandwidth required for this transmission? (Assume that the signal's peak to mean ratio is 20 dB.) [480 kbit/s, 240 kHz]

5.9. Information is to be transmitted as a linearly quantised, 8-bit, signal over a noisy channel. What probability of error can be tolerated in the detected PCM bit stream if the reconstructed information signal is to have a SNR of 45 dB? [4.1×10^{-6}]

5.10. Show that the peak signal to quantisation noise ratio, $(SN_qR)_{peak}$, for an n-bit, μ-law companded, communications system is given by:

$$(SN_qR)_{peak} = \frac{3 \times 2^{2n}}{[\ln(1 + \mu)]^2}$$

Hence calculate the degraded output SNR in dB for large signal amplitudes in an $n = 8$ bit companded PCM system with typical μ value when compared with a linear PCM system operating at the same channel transmission rate. [10.1 dB]

5.11. An 8-bit A-law companded PCM system is to be designed with piecewise linear approximation as shown in Figure 5.26. 16 segments are employed (with 4 co-linear near the origin) and the segments join at 1/2, 1/4, etc. of the full scale value, as shown in Figure 5.26. Calculate the approximate SNR for full scale and small signal values.
[Full scale 36.3 dB, small signal 71.9 dB]

5.12. A DM communication system must achieve a SNR of 25 dB. The DM step size is 1.0 V. The (zero mean) baseband information signal has a bandwidth of 3.0 kHz and a Gaussian pdf with an RMS value of 0.2 V. Use Figure 5.34 to estimate both the optimum sampling frequency and the gain of the amplifier which is required immediately prior to the modulator input. [96 kHz, 10.5 dB]